Biological Distributed Algorithms and Lévy Walks

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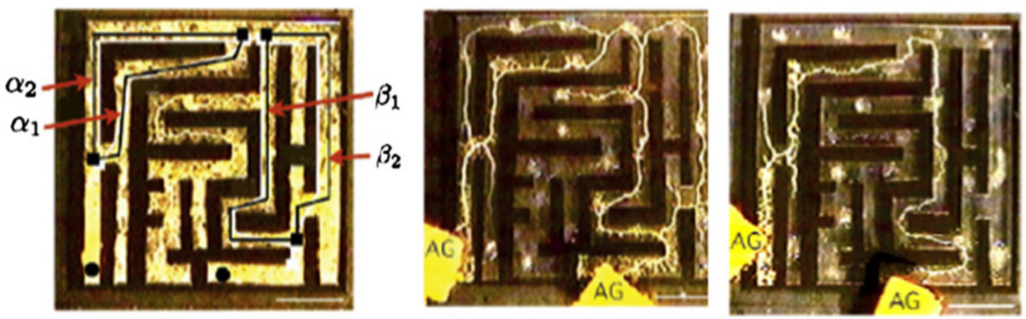
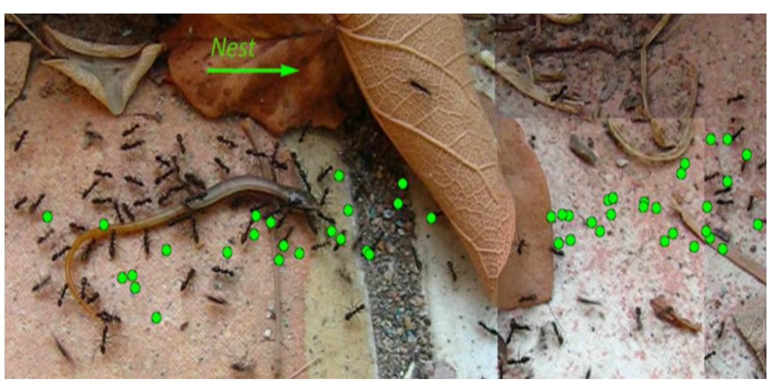
Credits: Slides largely adapted from previous ones by F. D'Amore

Biological Distributed Algorithms

How do flocks of birds synchronize their flight? [Chazelle '09]



How does Physarum polycephalum finds shortest paths? [Mehlhorn et al. 2012-...]



How ants perform collective navigattion? How do they decide where to relocate their nest?

What are Lévy Walks?

Lévy walk. Random walk with i.i.d.
- uniform directions
- power-law step-lengths: for fixed $\alpha > 1$, for each $d \in \mathbb{R}$

$$f(d) \sim 1/d^{\alpha}$$

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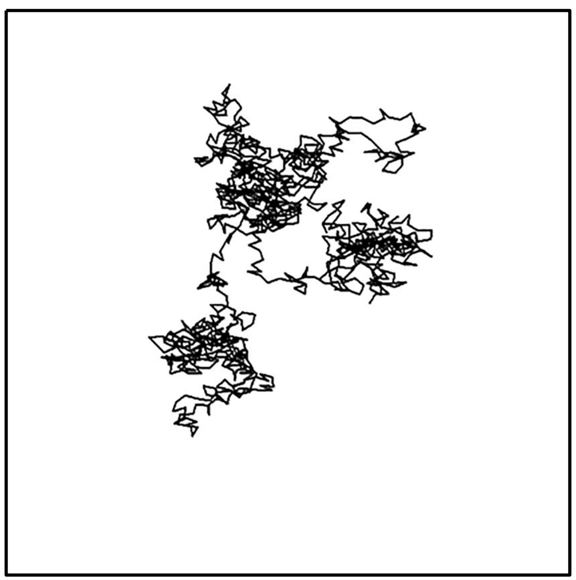
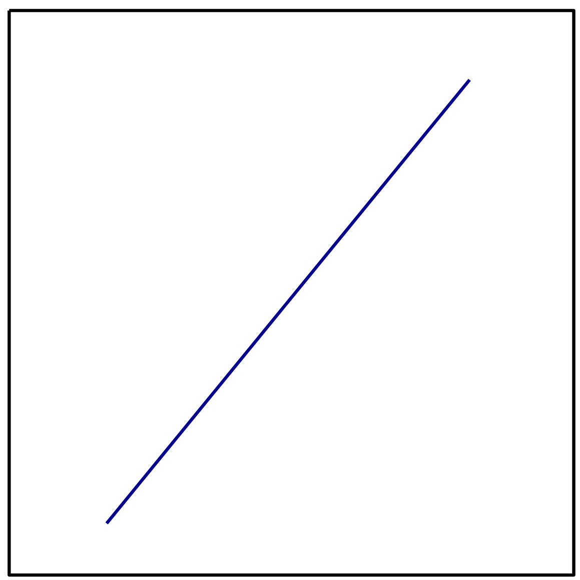
normal diffusion
(random walk/brownian motion)



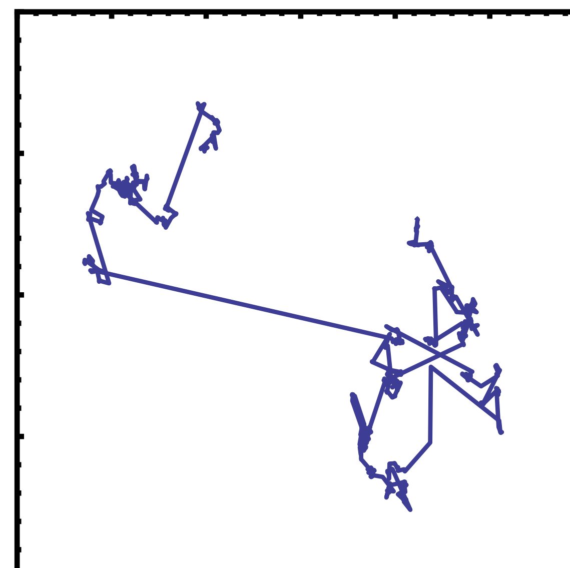
$3 \leq \alpha$

ballistic diffusion
(straight/ballistic walk)



$1 < \alpha \leq 2$

super diffusion
(between (a) and (b))

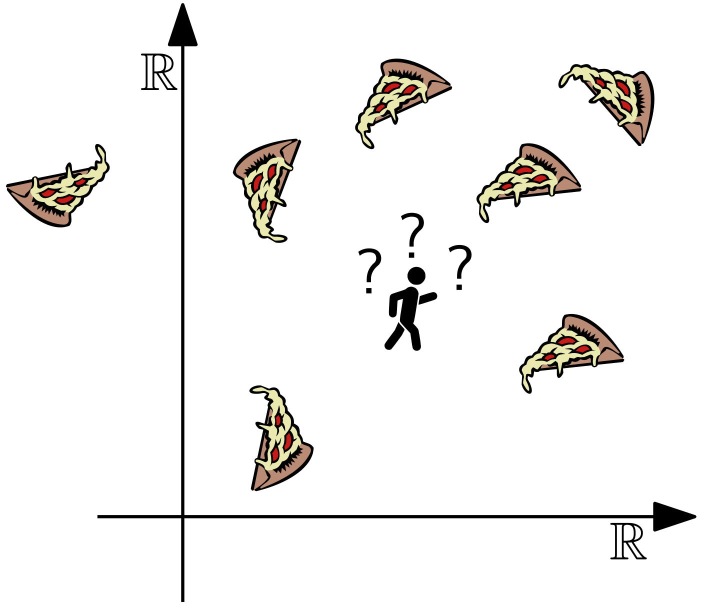


$2 < \alpha < 3$

Optimality of Lévy Walk

Formal scenario:

- a density distribution ρ in $\mathbb{R}^n$ describing food locations
- an uninformed walker searching for food in $\mathbb{R}^n$



Question: which strategy maximizes the expected food discovery rate (number of discovered food locations over travelled distance)?

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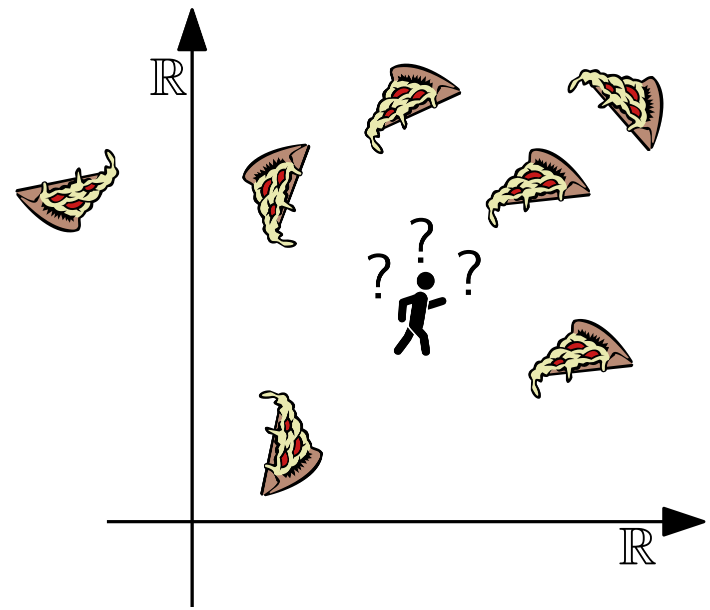

Question: which strategy maximizes the expected food discovery rate (number of discovered food locations over travelled distance)?

Result: in order to maximize the expected food discovery rate, the walker should perform the Lévy walk with exponent $\alpha = 2$.

The ANTS Problem

[*Korman et al.*, PODC, '12] introduces the Ants Nearby Treasure Search (ANTS) Problem

Setting:

- k (mutually) independent walkers (agents) start moving on $\mathbb{Z}^2$ from the origin
- time is synchronous and marked by a global clock
- one special node $\mathcal{T} \in \mathbb{Z}^2$, the *treasure*, at (Manhattan) distance ℓ from the origin

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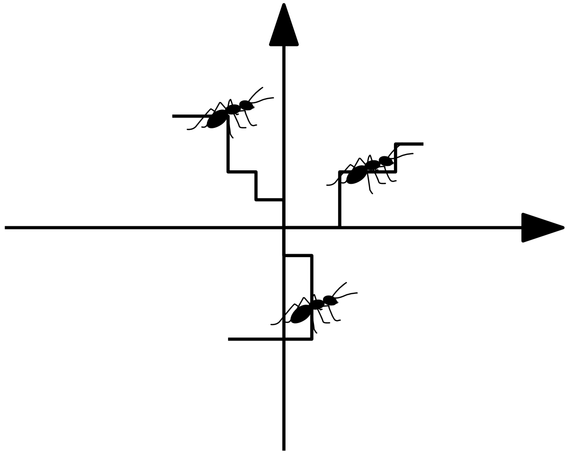
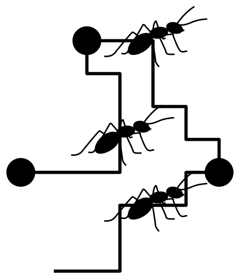
Question: which strategy is the best one to find the treasure?

Preliminaries: Total Work

Definition (*work*): k agents moving for t steps make a work equal to $k \cdot t$





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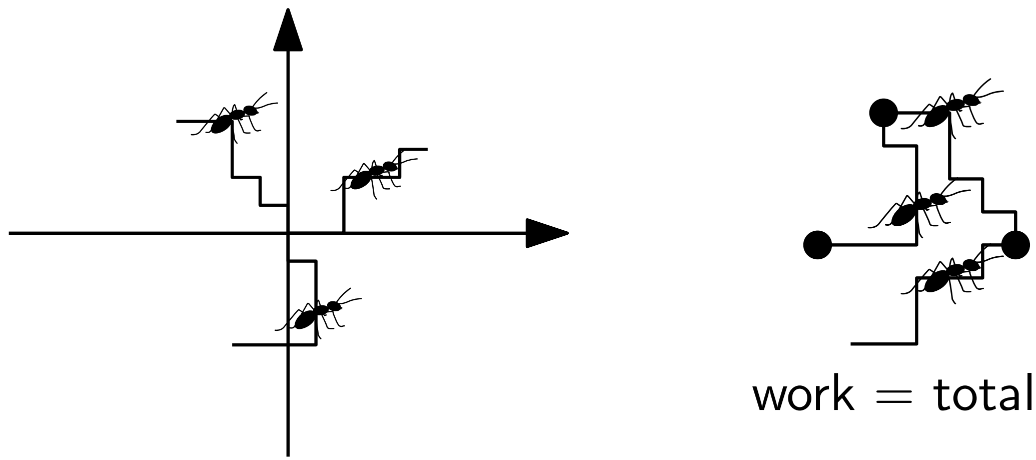


work $=$ total path covered

Lemma [*Korman et al.*, PODC, '12]. Locate $\mathcal{T}$ u.a.r. at Manhattan distance at most ℓ. For any $k \geq 1$, and for any search algorithm, the required work to find $\mathcal{T}$ is $\Omega\left(\ell^2\right)$, both with constant probability and in expectation.

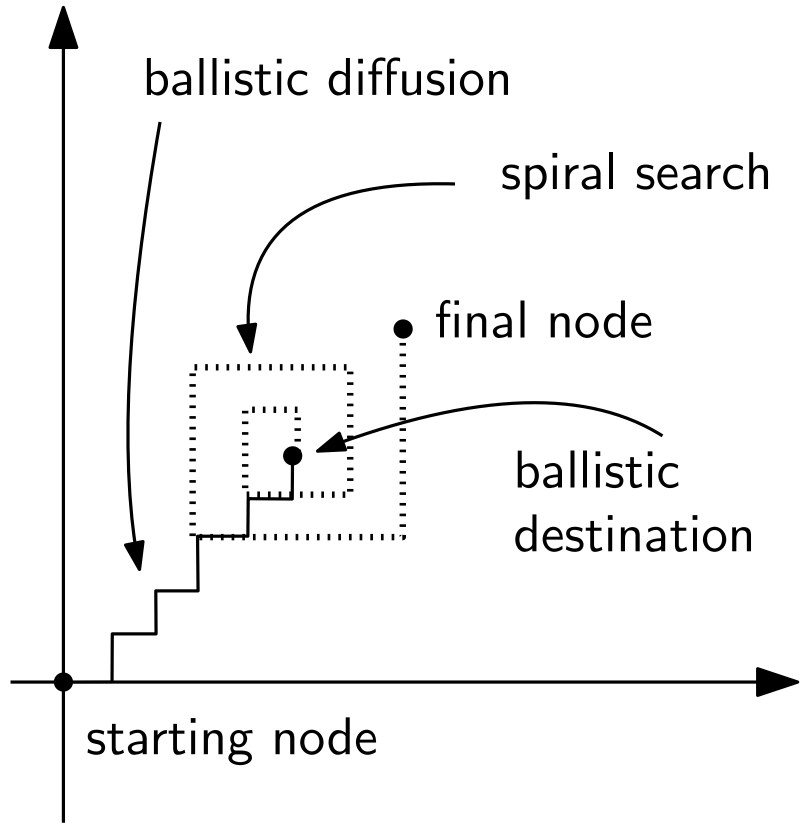
The Harmonic Search Algorithm

[*Korman et al.*, PODC, '12] proposes a search algorithm which is almost optimal and which is *natural*

($\alpha > 1$ fixed)

The Harmonic Search algorithm:

(a) sample a jump-length d with prob c_α / d^α
(b) (ballistic diffusion) in d steps, move to a destination at distance d chosen u.a.r.
(c) (normal diffusion) start a spiral search for $d^{\alpha+1}$ steps
(d) return at the origin and repeat from (a)



Our Results: Analysis of Discerete Lévy Walks

Theorem.

k parallel Lévy walks with exponent $\alpha \in (2,3)$ start from origin.
Target at distance ℓ, s.t. $(\ln \ell)^c \leq k \leq \ell$ (universal c).

Let $\alpha^* = 3 - \frac{\ln k}{\ln \ell}$.

With prob. going to 1 w.r.t. k and ℓ:

- If $\alpha \simeq \alpha^*$, then at least one walk finds target within $(\ell^2/k) \cdot (\ln \ell)^c$ steps.
- If $\alpha \gtrsim \alpha^\star$, then no walk finds target within $(\ell^2/k) \cdot \ell^\beta$ steps, where $\beta \simeq (\alpha - \alpha^*)/2$.
- If $\alpha \lesssim \alpha^\star$, then target is never found, by any walk.

Optimal, Simple Algorithm for the ANTS Problem

Optimal solution to the ANTS problem (up to polylogs):

Each agent chooses uniformly and independently at random an exponent $\alpha \in (2,3)$ and then performs a Lévy walk with parameter α.

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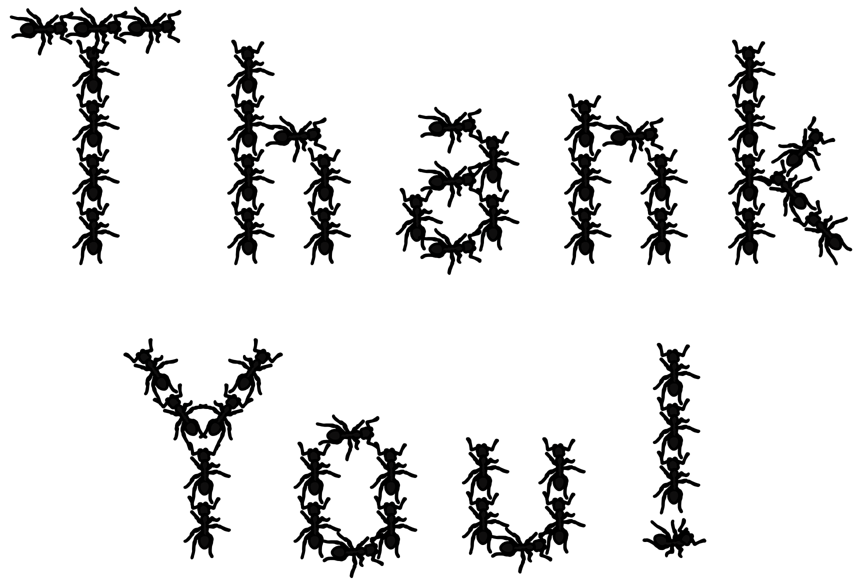
Key points of Lévy Walk project.

- Definition of **discrete version** of the Lévy walk
- Analysis of k Lévy walks for the ANTS Problem
- For any $\alpha > 1$ there is a k s.t. k Lévy walks achieve optimal work
- The exponent $\alpha = 2$ is not so "universal" !
- Optimal and natural algorithm for the ANTS Problem


Thank You!