Computing through Simplicity: Computational Dynamics and Applications

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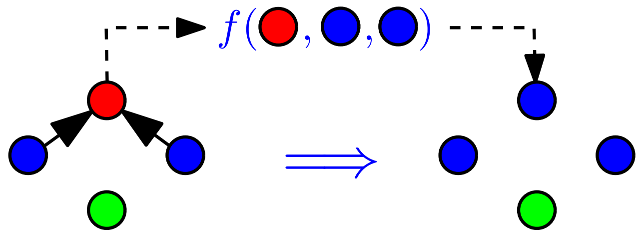
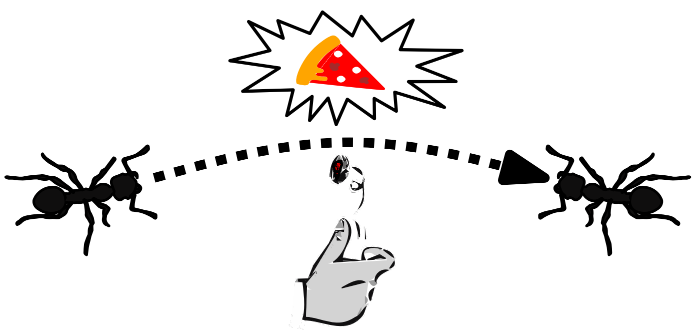
Research Directions

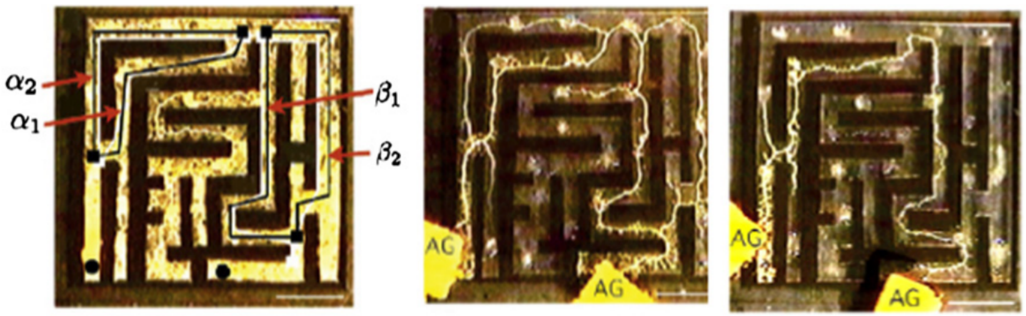
- **Computational Dynamics.**
  Achieving simplicity in randomized distributed algorithms.



- **Biological Distributed Algorithms.**
  Going into biology and back, through the algorithmic lens (Natural Algorithms).

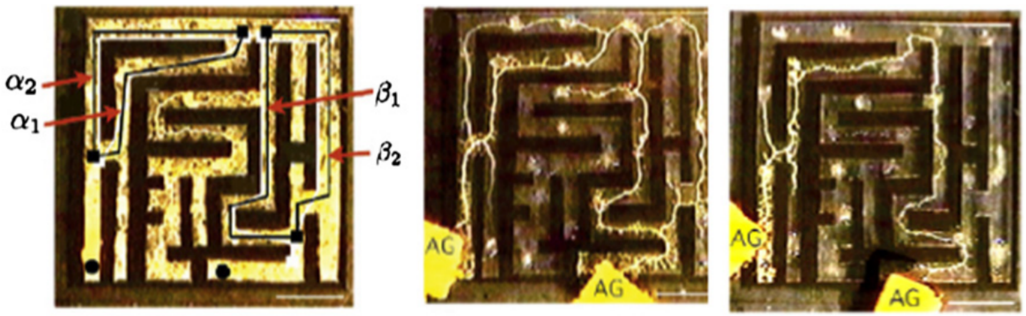


Natural Algorithms



How does Physarum polycephalum
finds shortest paths? [Mehlhorn et al. 2012-...]

Natural Algorithms



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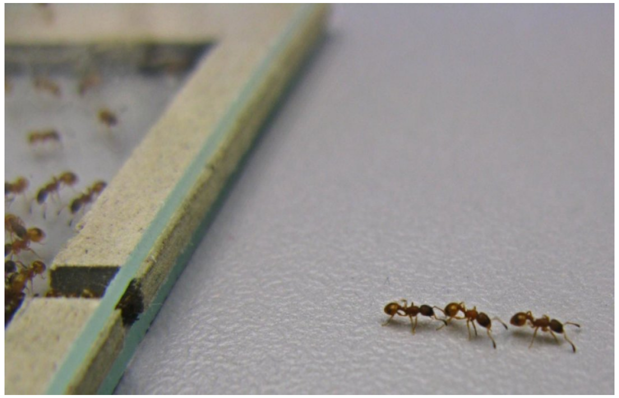
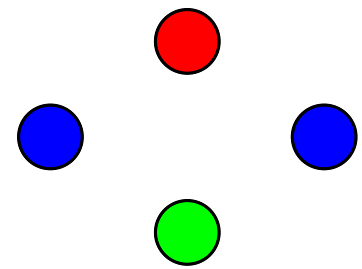
How ants perform collective navigattion? How do they decide where to relocate their nest?



Computational **Dynamics**

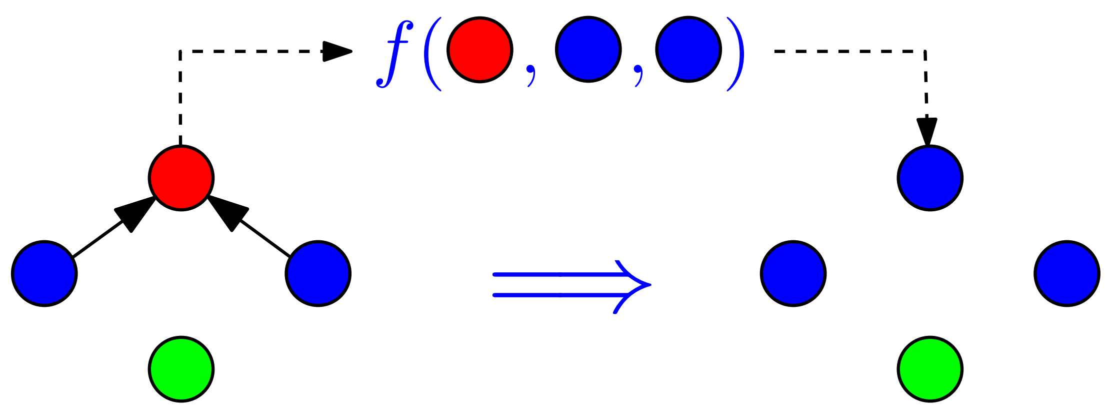
Anonymous agents

- small set of possible states
- *simple* update function f



At each step:

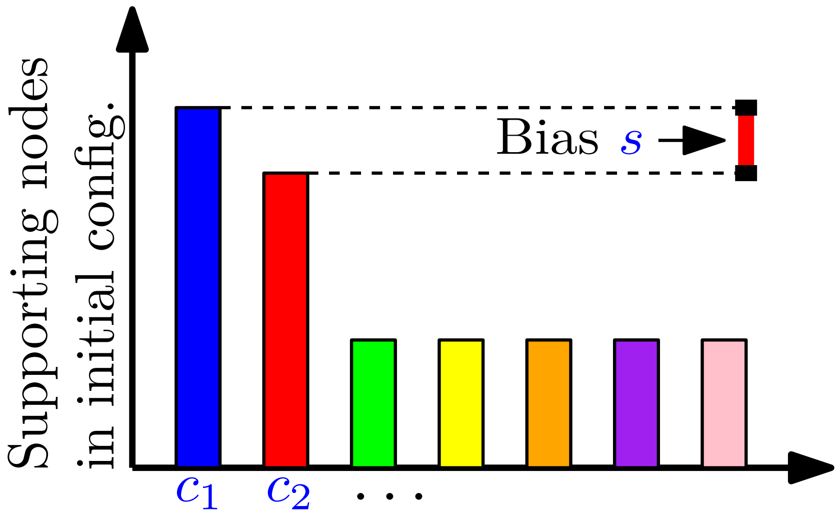
Update depends on states of random subset of agents



Dynamics for Plurality Consensus

Plurality Consensus.

- Each agent initially has a value in $\{1, ..., k\}$.
- There is a small initial **bias** (majority − 2nd-maj. color).
- Each agent eventually has the most frequent initial value.



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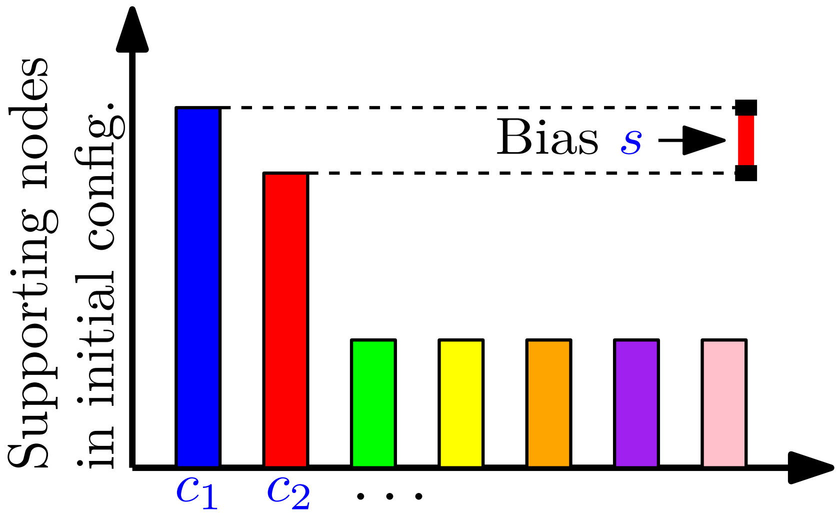
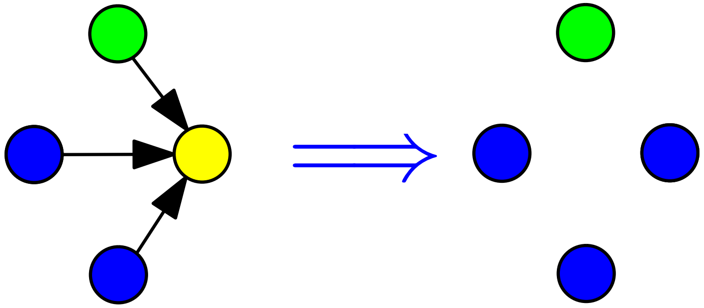


3-Majority Dynamics.

At each round, each agent samples 3 agents in the system and adopts the majority color.

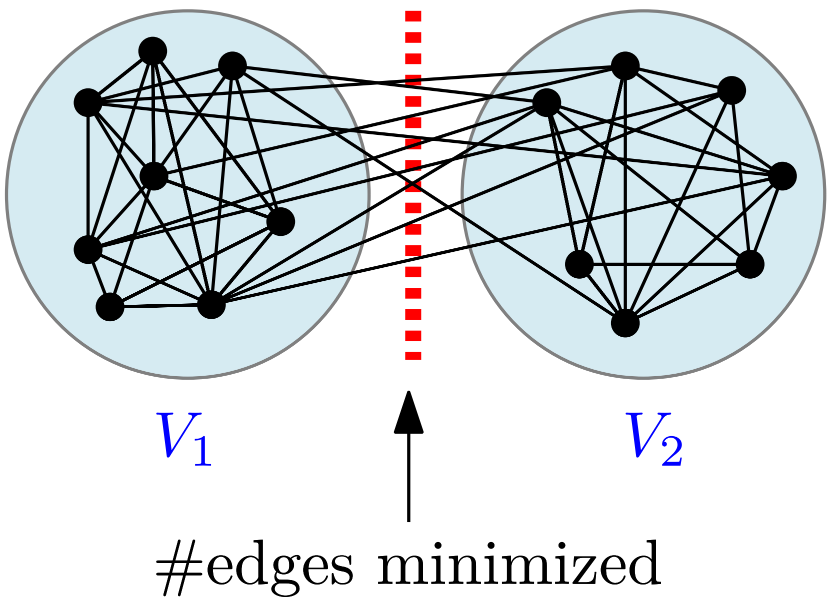
Theorem.

3-Majority Dynamics converges to plurality in $\mathcal{O}(k \log n)$ rounds



Clustering

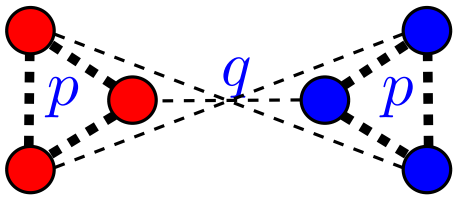
Minimum Bisection Problem.
Find balanced bipartition $|V_1| = |V_2|$ that minimizes cut.



[Garey et al. '76]: Minimum bisection problem is NP-Complete!

Stochastic Block Model (SBM)

- "Communities" V_1, V_2, with $|V_1| = |V_2|$.
- include each edge with probability
  - p if edge inside V_1 or V_2,
  - q if edge between V_1 and V_2.



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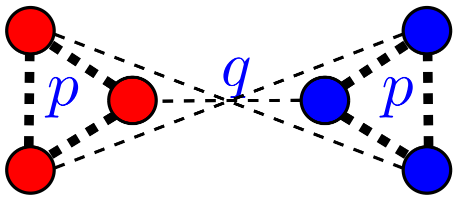

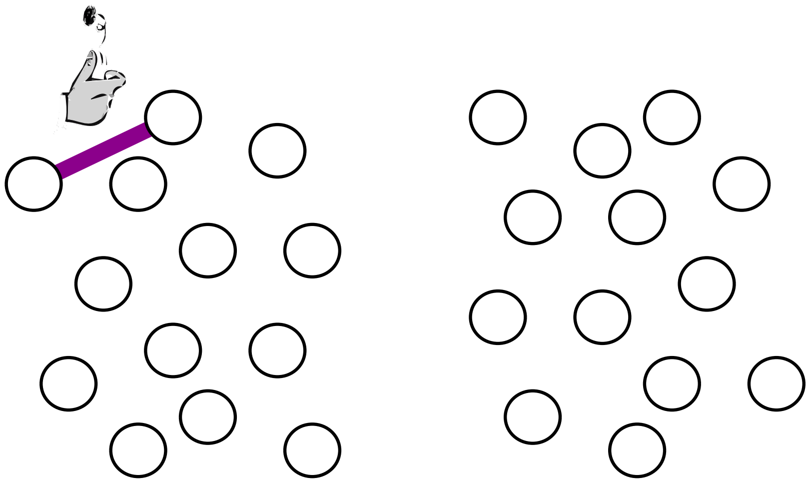

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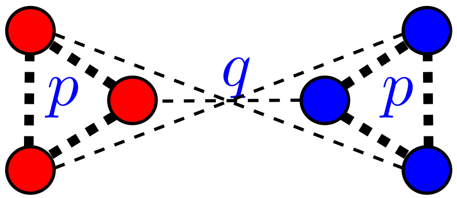

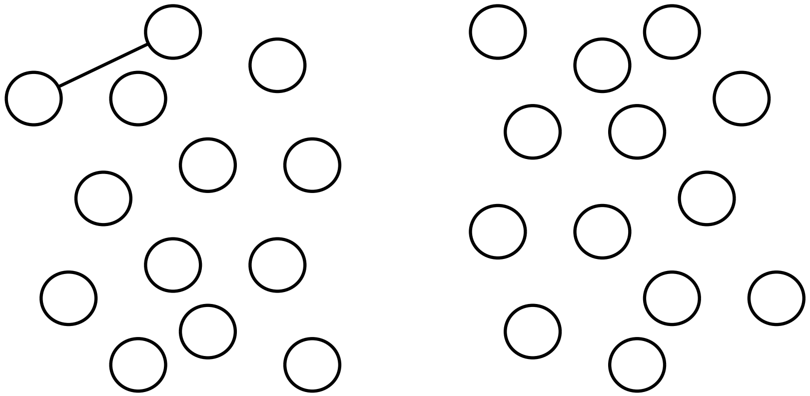

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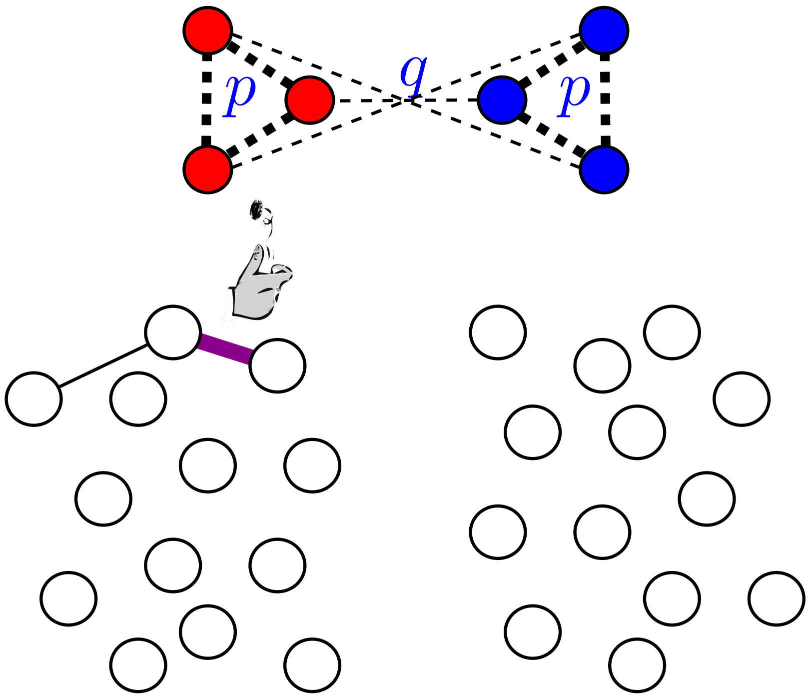

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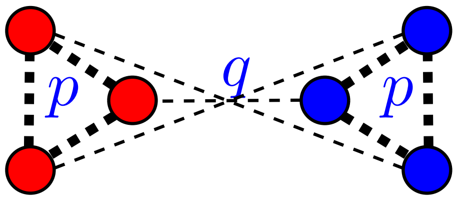

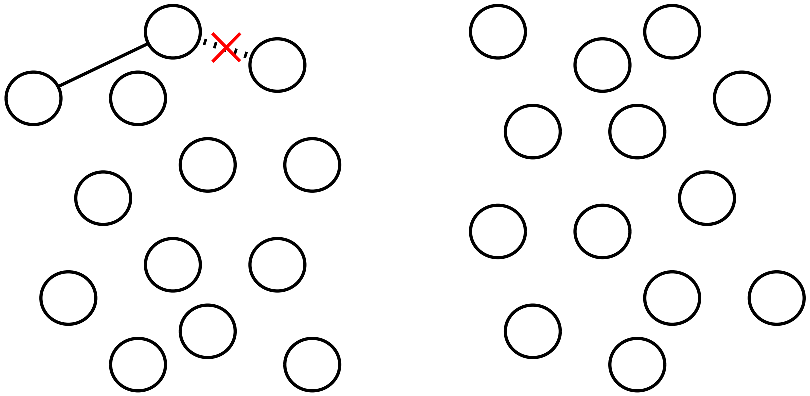

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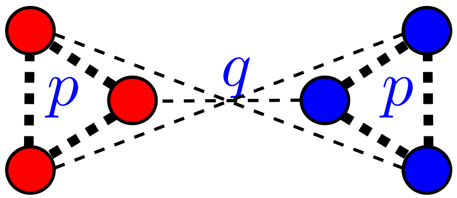

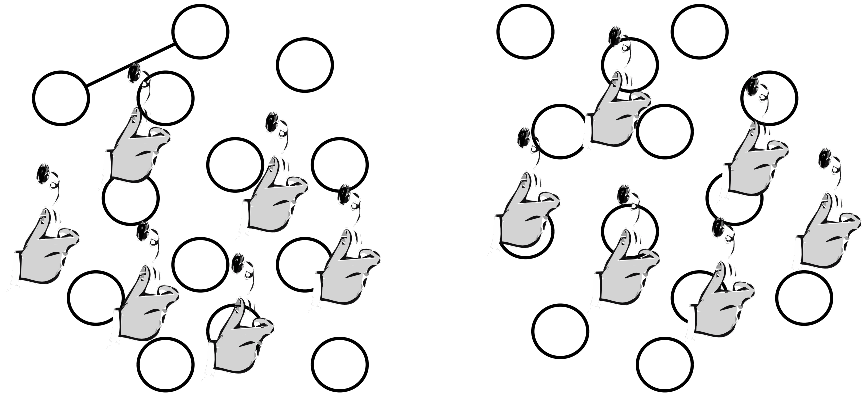

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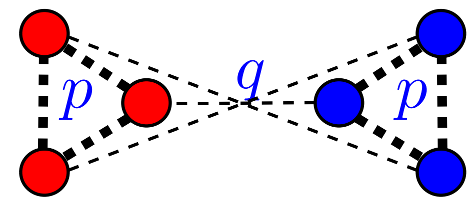

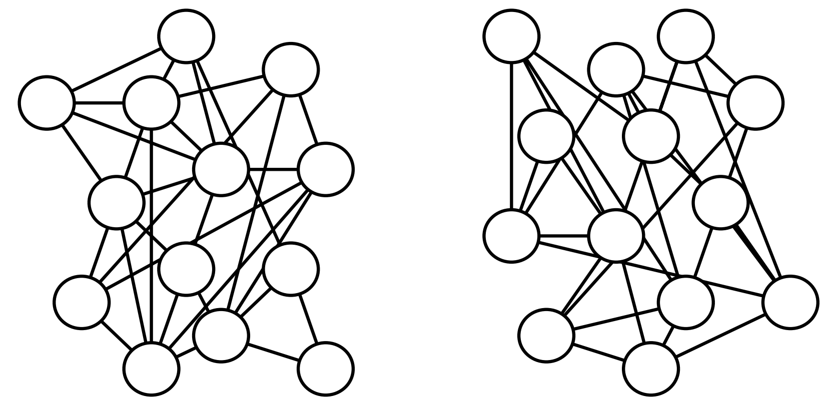

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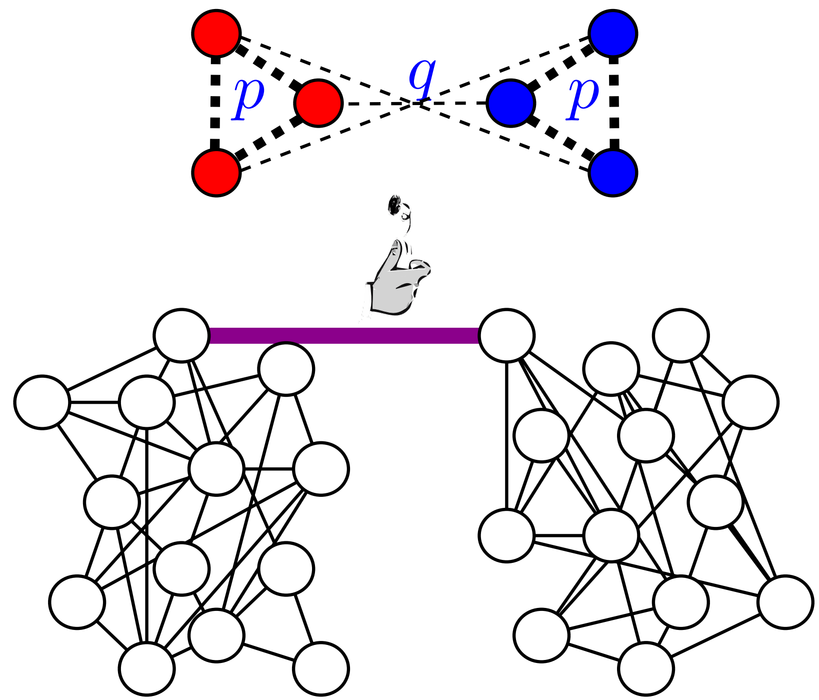

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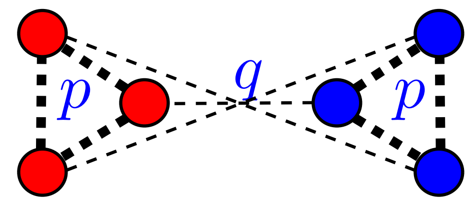

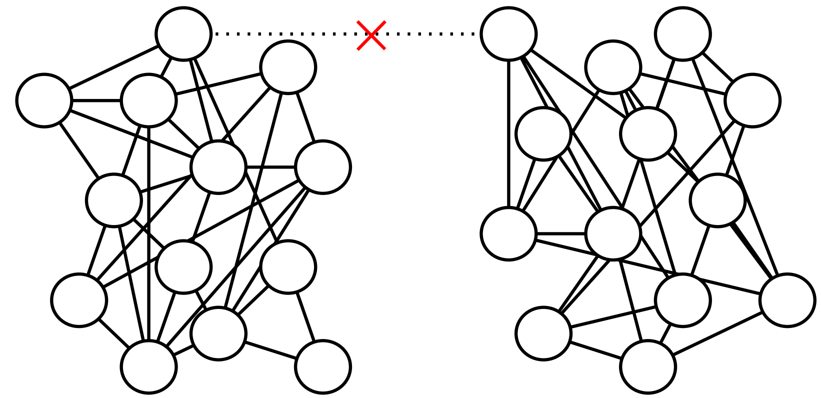

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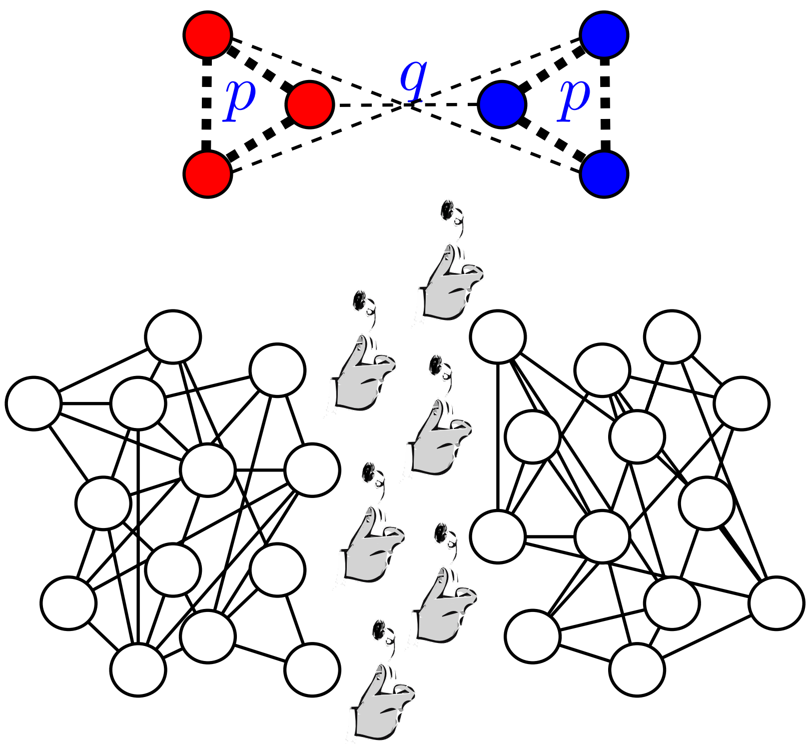

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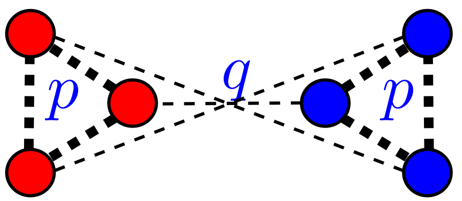

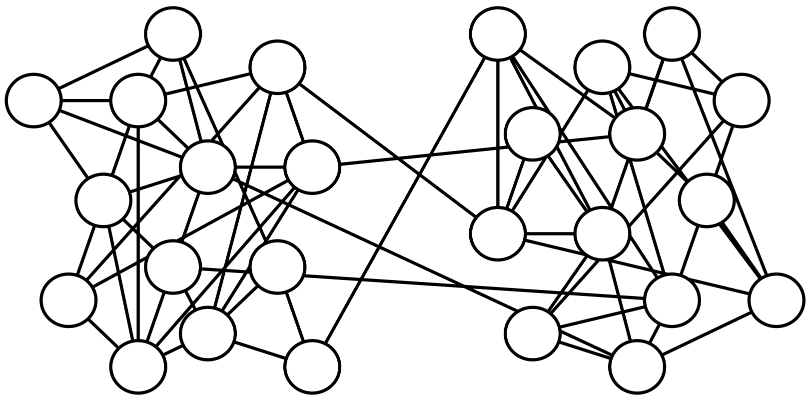

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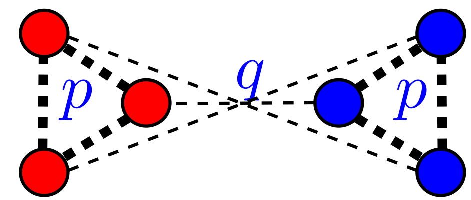
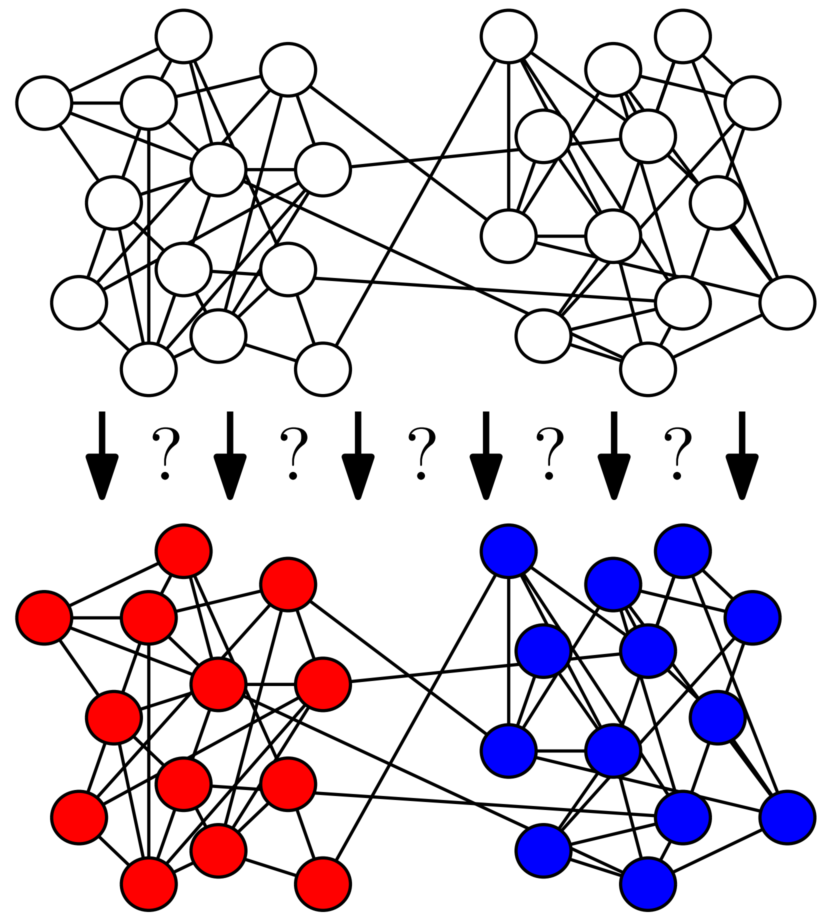


"Reconstruction" problem. Given graph generated by SBM, find original clusters.



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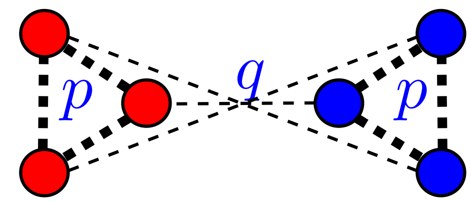

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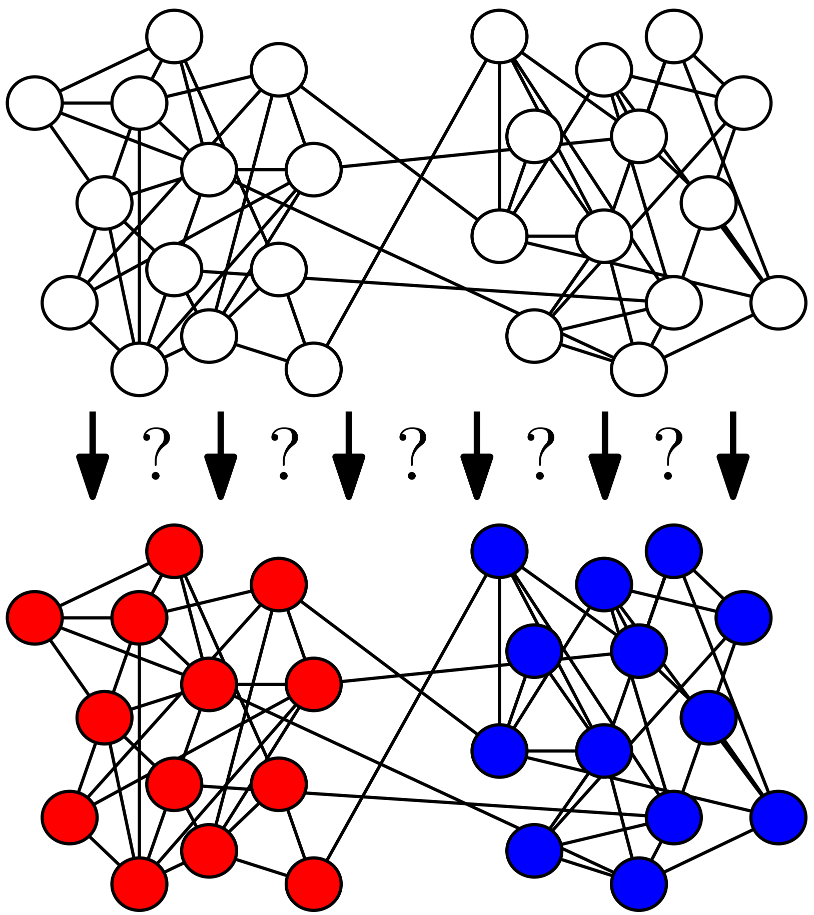
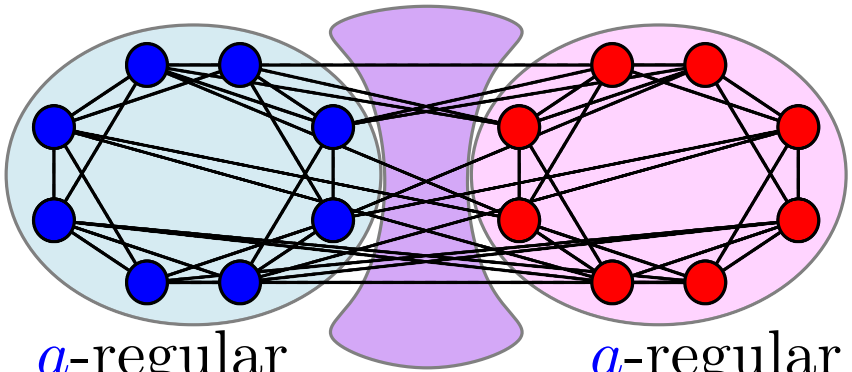


Theorem. [Mossel et al. 2012-] Clustering possible **if and only if** p and q in a precise regime.

Clustering with **Averaging Dynamics**

Regular Stochastic Block Model:



a-regular a-regular

b-regular bipartite

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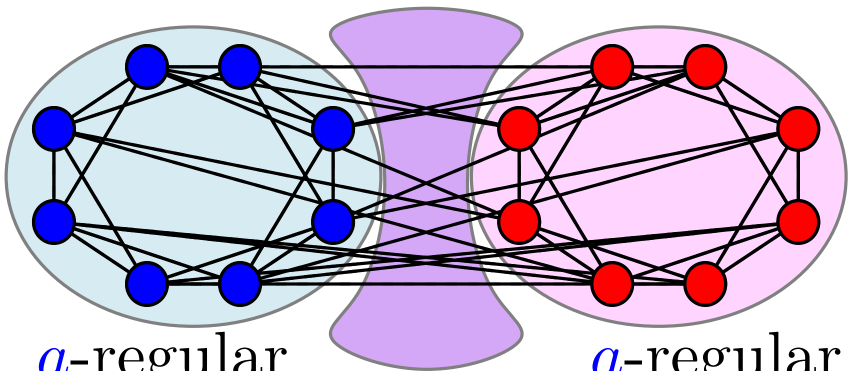
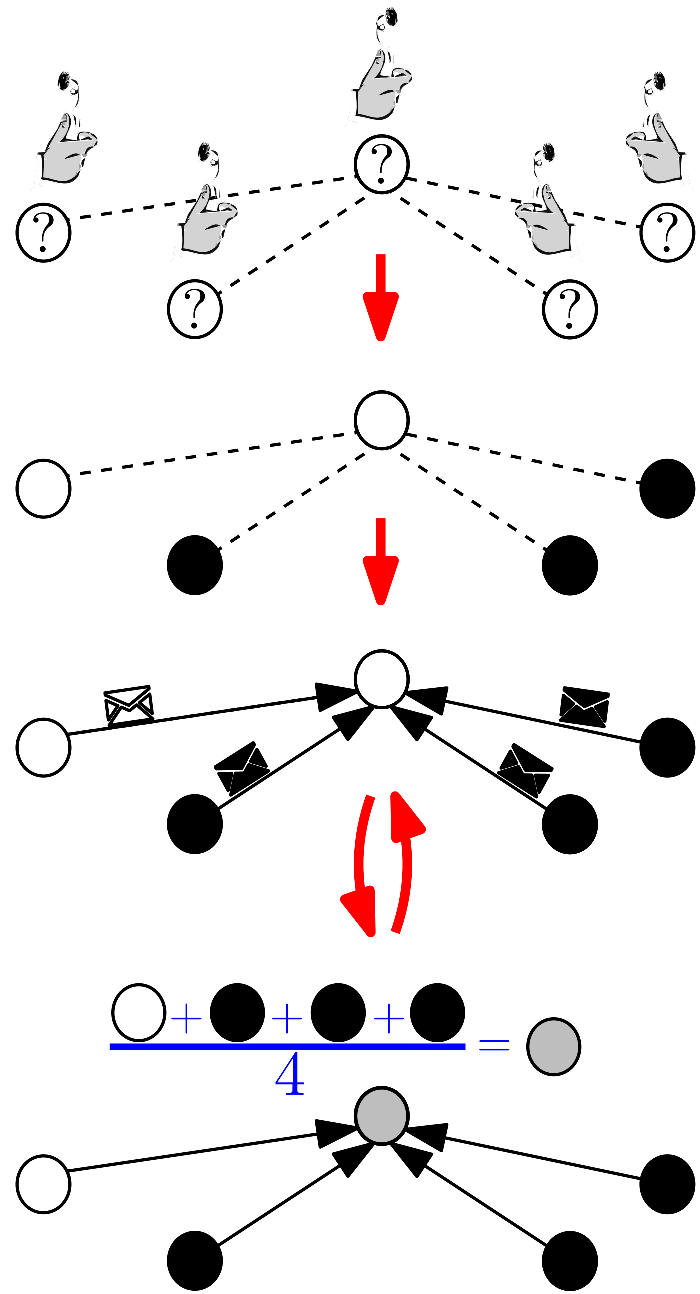


a-regular $\quad$ a-regular

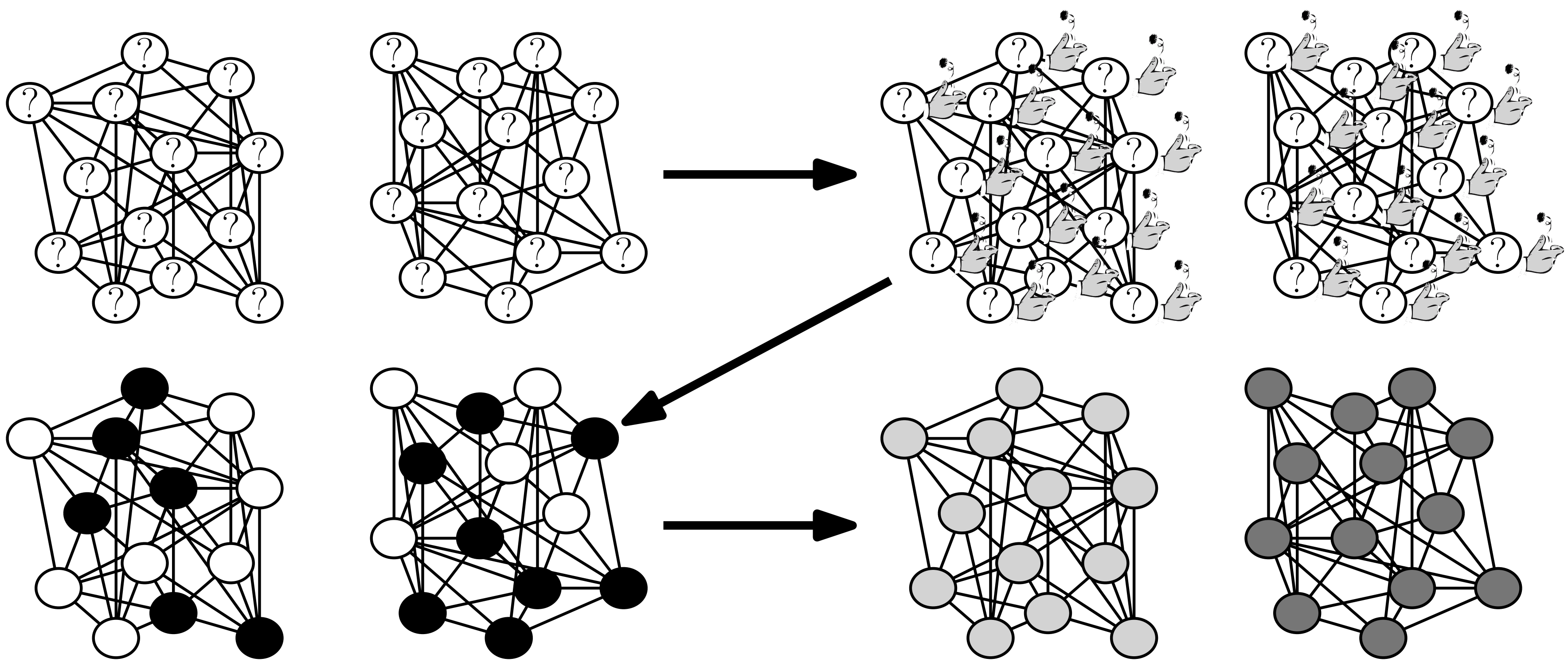
b-regular bipartite

All nodes at the same time:

- At $t = 0$, randomly pick value $x^{(t)} \in \{\mathbf{+1}, \mathbf{-1}\}$
- Then, at each round set value $x^{(t)}$ to average of neighbors



Why it Works: Intuition



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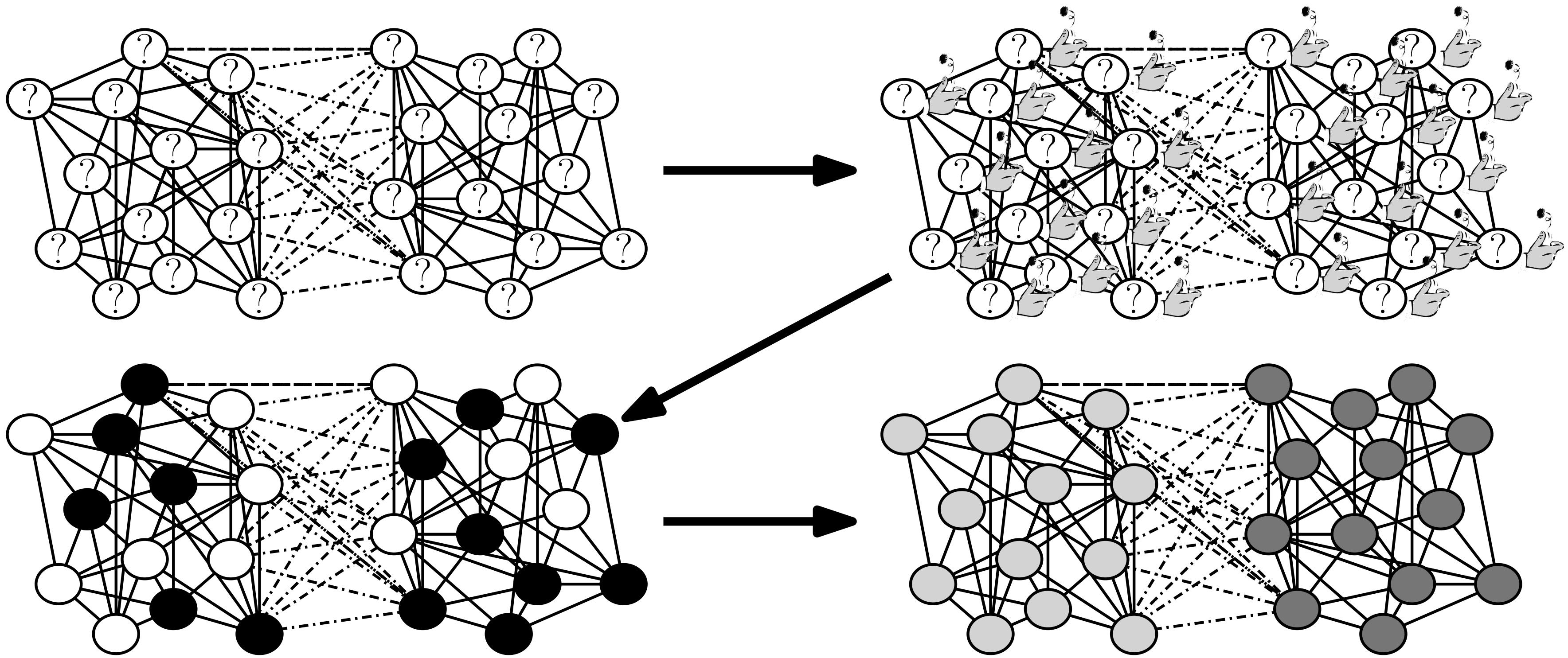

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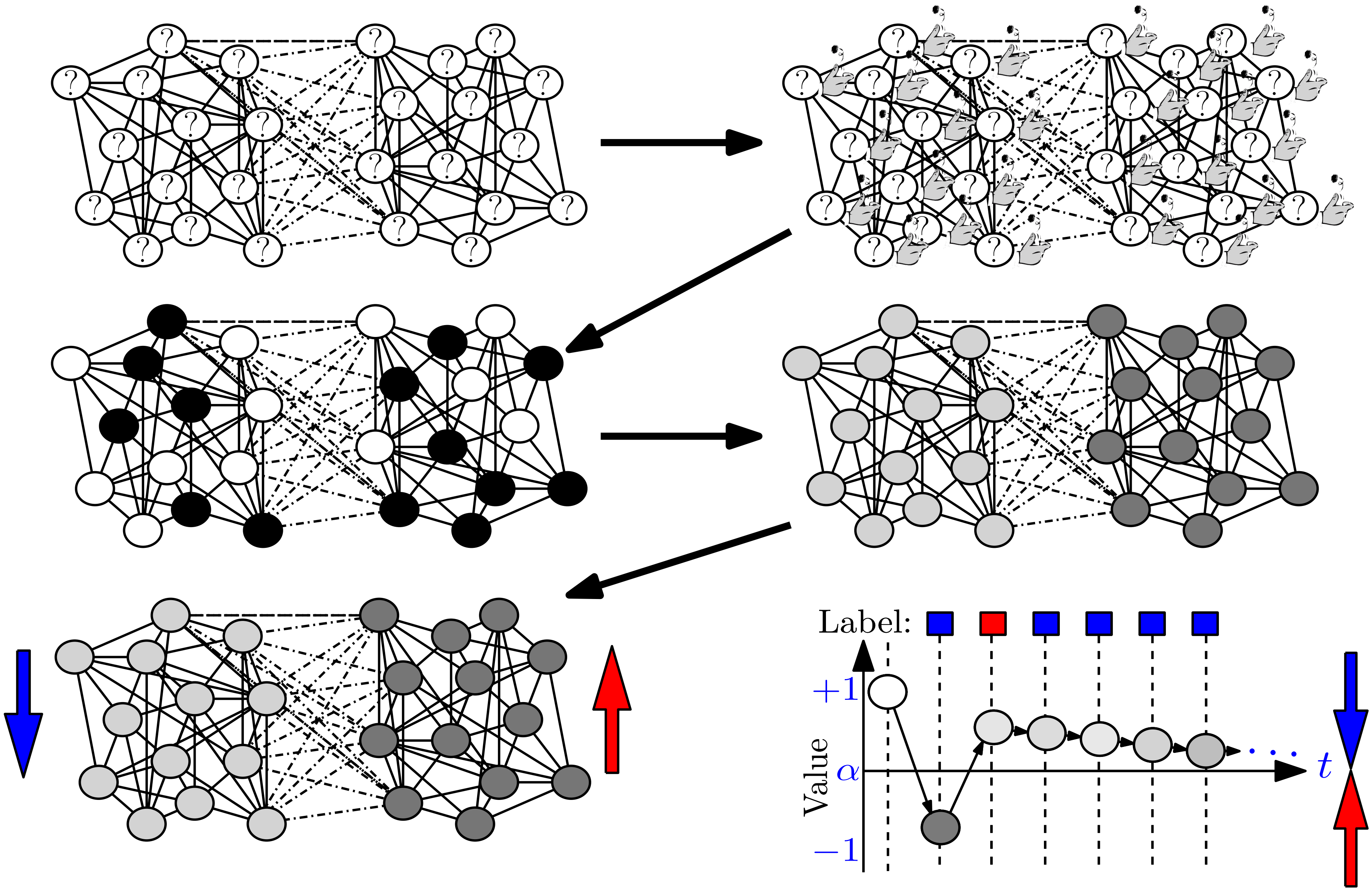
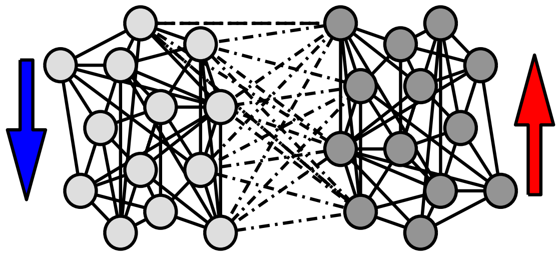


- Set label to **blue** if $x^{(t)} < x^{(t-1)}$, **red** otherwise

Why It Works: Proof Idea

Theorem. In Regular Stochastic Block Model with $a - b > \sqrt{2(a+b)}$, Averaging Dynamics finds clusters after $\frac{\log n}{\log \lambda_2/\lambda_3}$ steps with high probability.



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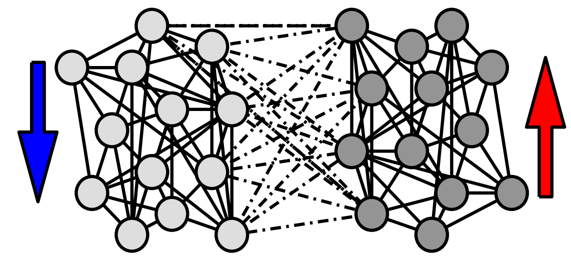
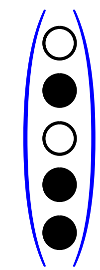


Averaging is a **linear** dynamics:

$$\mathbf{x}^{(t)} = P \cdot \mathbf{x}^{(t-1)} = P^t \cdot \mathbf{x}^{(0)}$$

P transition matrix of random walk on G and $\mathbf{x}^{(t)} =$ 

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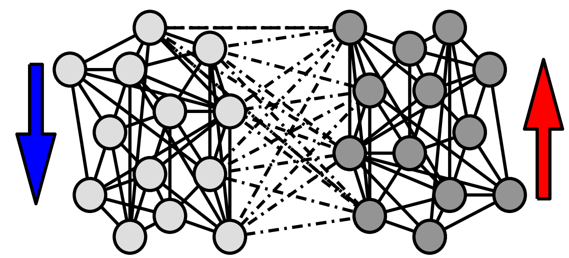

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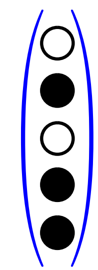
P transition matrix of random walk on G and $\mathbf{x}^{(t)} =$ 

$$\mathbf{x}^{(t)} = \frac{1}{\tilde{\Theta}(\sqrt{n})} \begin{pmatrix} 1 \\ \vdots \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} + \left(\frac{a-b}{a+b}\right)^t \frac{1}{\tilde{\Theta}(\sqrt{n})} \begin{pmatrix} 1 \\ \vdots \\ 1 \\ -1 \\ \vdots \\ -1 \end{pmatrix} + \mathbf{e}^{(t)}$$

negligible after $t \gg \frac{\log n}{\log \lambda_2/\lambda_3}$

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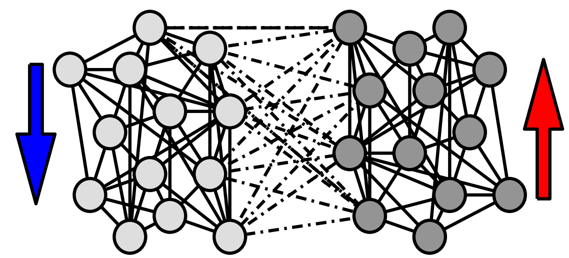

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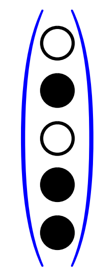

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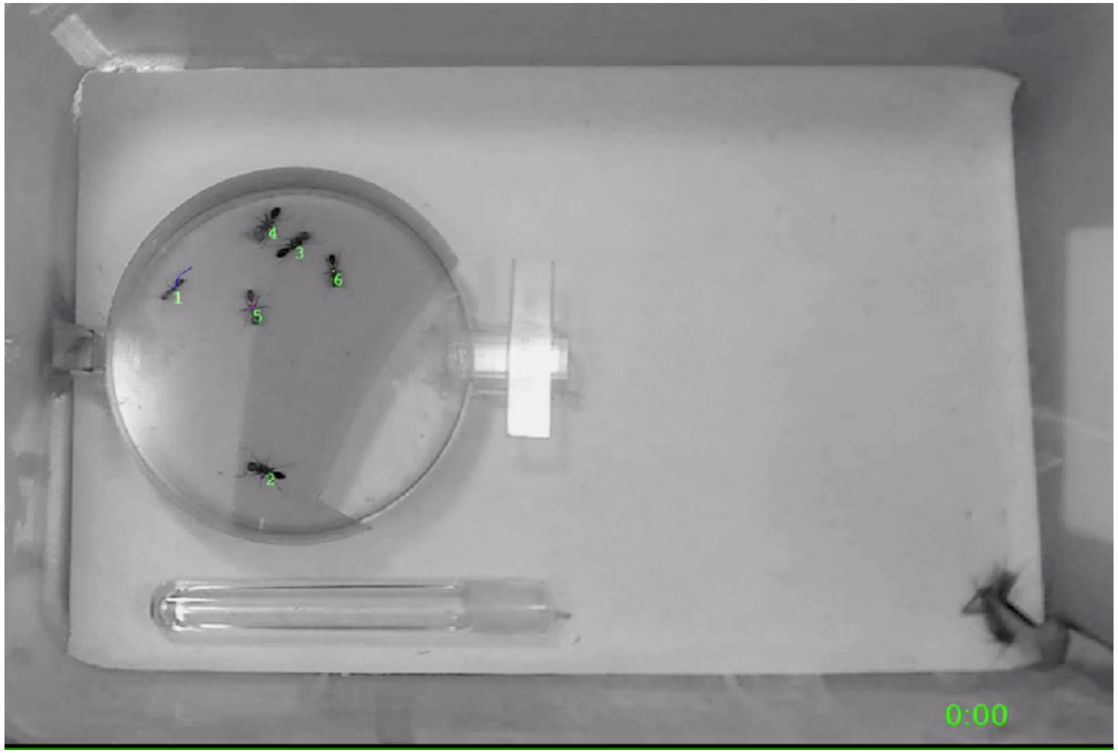
negligible after $t \gg \frac{\log n}{\log \lambda_2/\lambda_3}$

$$\mathrm{sign}(\mathbf{x}^{(t)}(u) - \mathbf{x}^{(t-1)}(u)) = \mathrm{sign}\left(\begin{pmatrix} 1 \\ \vdots \\ 1 \\ -1 \\ \vdots \\ -1 \end{pmatrix}\right)$$

Example of Research on Collective Behavior

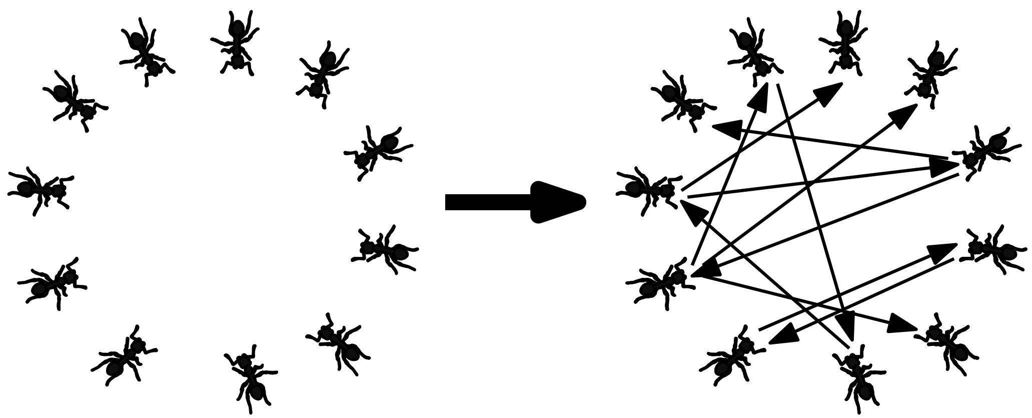
Recruitment in Desert Ants



Cataglyphis niger needs to recruit nest mates to carry food.
Data suggest that they communicate by simple, *stochastic noisy interactions*.
We provide **mathematical evidence** on why stochastic noisy interactions imply *small group size*.

Noisy & Stochastic Interactions

Stochastic Interactions.
At each round, each agent receives a message from another random agent.



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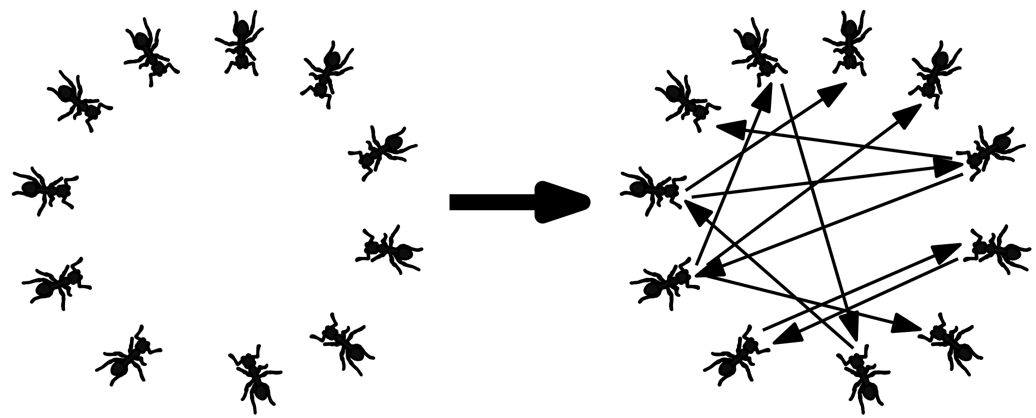
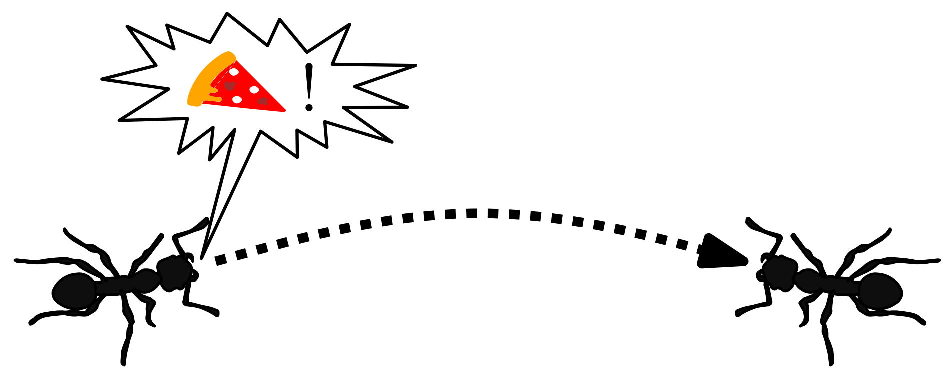


Noisy Communication.
Before being received, each bit is flipped with probability $1/2 - \epsilon_n$.



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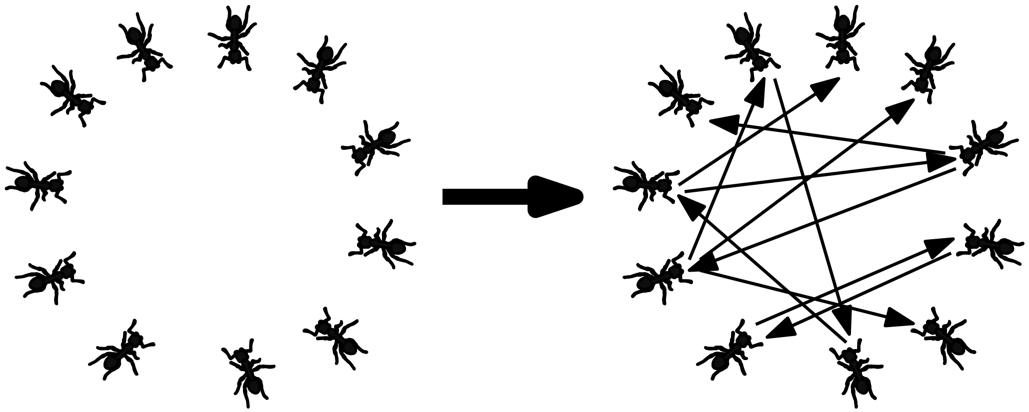

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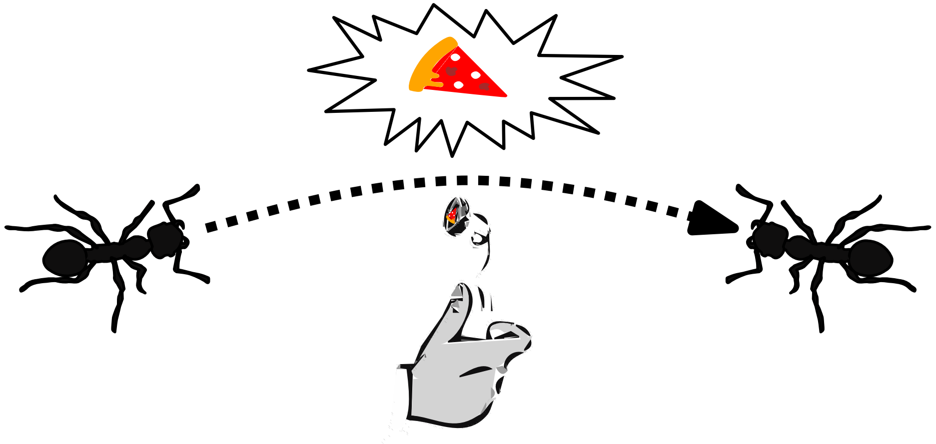

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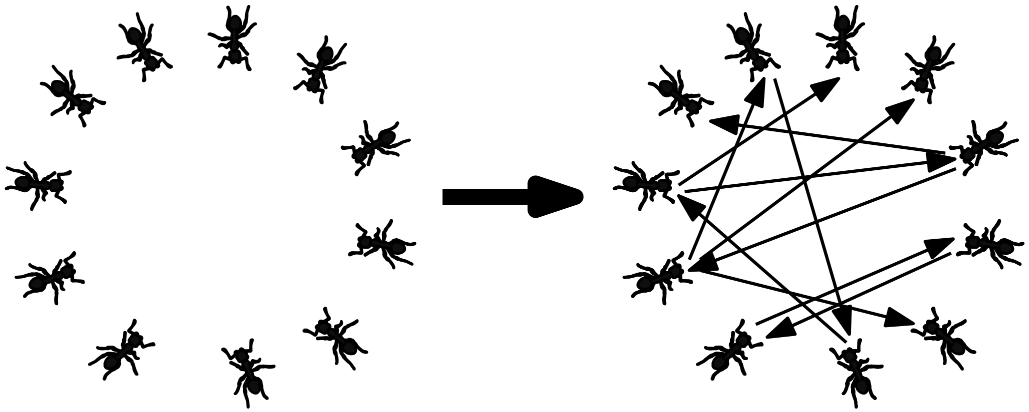
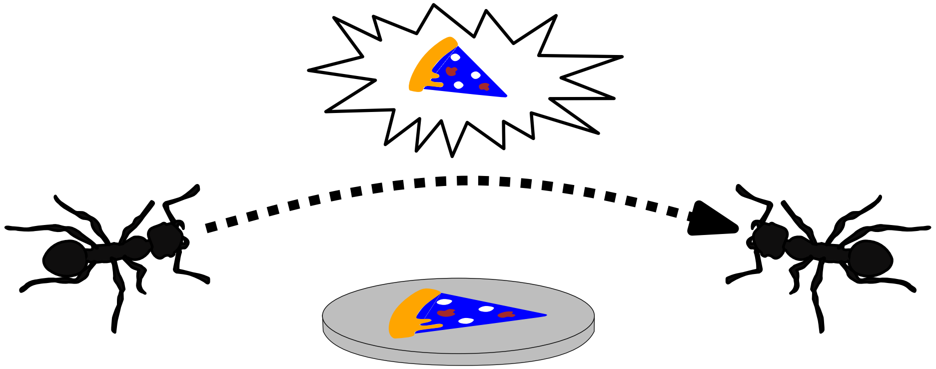


Noisy Communication.
Before being received, each bit is flipped with probability $1/2 - \epsilon_n$.



Noisy & Stochastic Interactions

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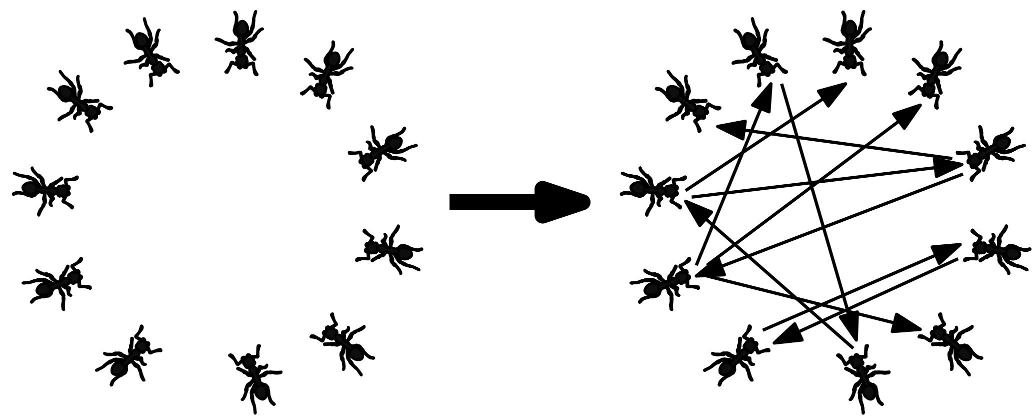

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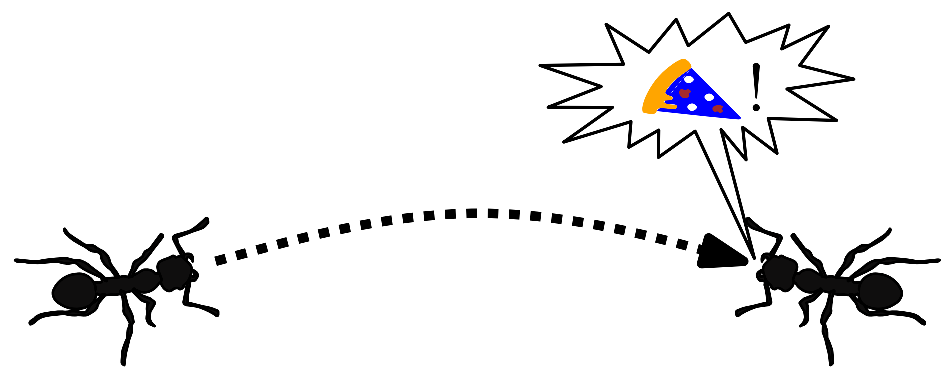
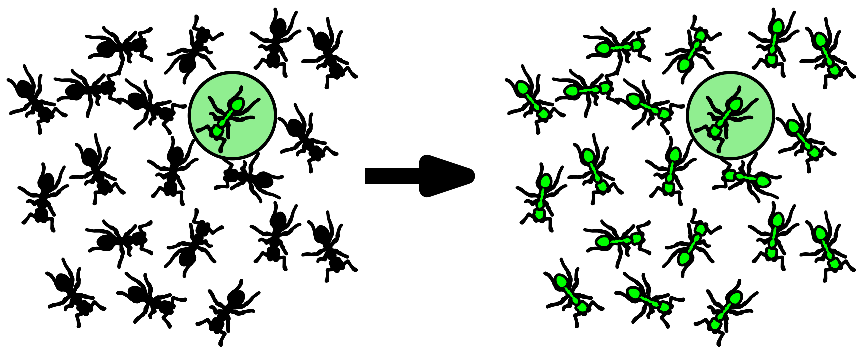
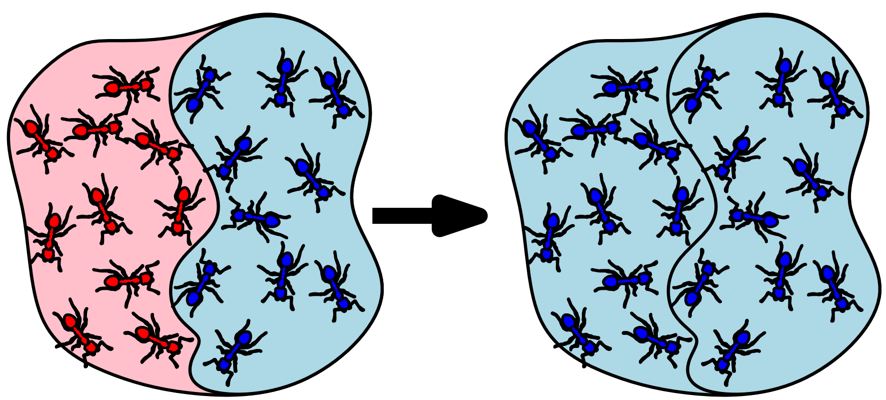


Noisy vs Noiseless Broadcast and Consensus



Broadcast. All nodes eventually receive the message of the source.

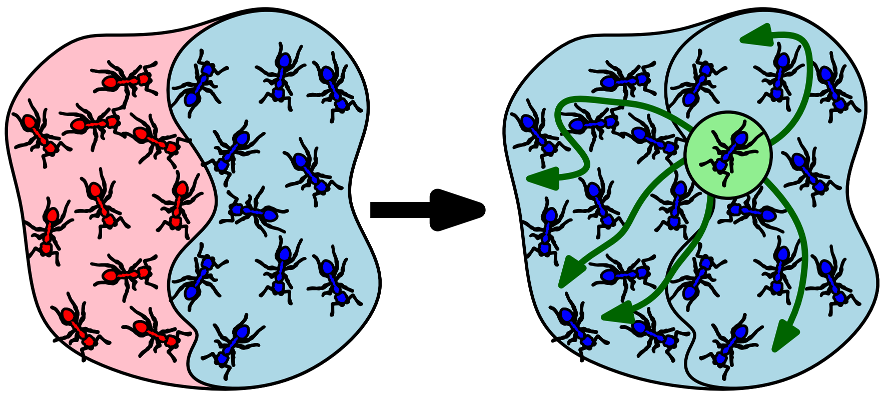


(Valid) Consensus. All nodes eventually support the value initially supported by one of them.

Reductions and Lower Bounds

Broadcast $\Longrightarrow$ Consensus

Noiseless Consensus
$\Longrightarrow$ **Noiseless**
(variant of) Broadcast



Noiseless Consensus and Broadcast are "*equivalent*"

Reductions and Lower Bounds

Broadcast $\Longrightarrow$ Consensus

Noiseless Consensus
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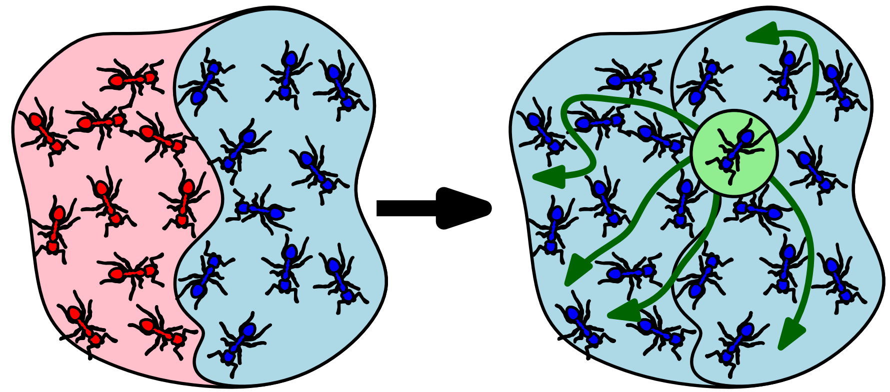
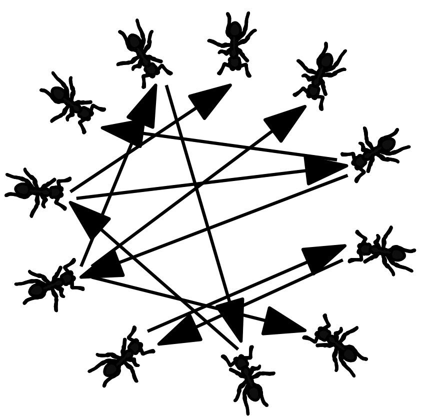
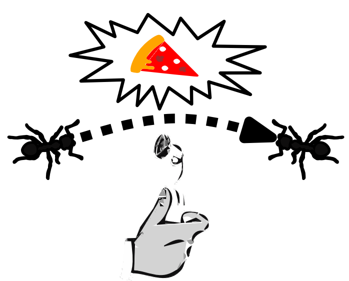


Noiseless Consensus and Broadcast are "*equivalent*"



+



$\Longrightarrow$

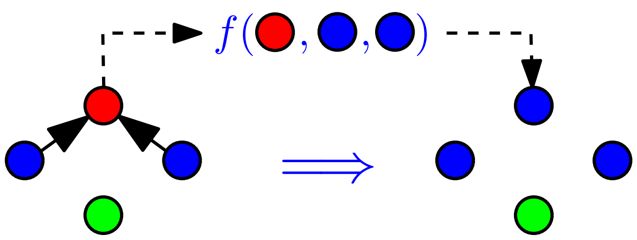
Noisy Consensus:
$\Theta(\frac{\log n}{\epsilon^2})$ rounds

Noisy Broadcast:
$\Theta(n \cdot \frac{\log n}{\epsilon^2})$ rounds

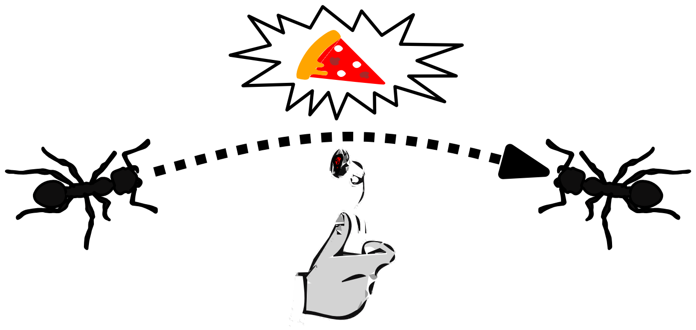
Noisy Broadcast is *exponentially harder*
than **Noisy** Consensus

Future Research Directions

- **Computational Dynamics.**
  Achieving simplicity in randomized distributed algorithms.



- **Biological Distributed Algorithms.**
  Going into biology and back, through the algorithmic lens (Natural Algorithms).



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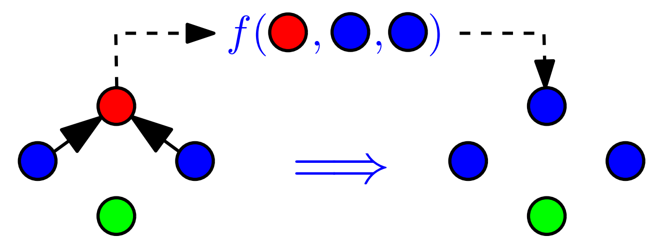

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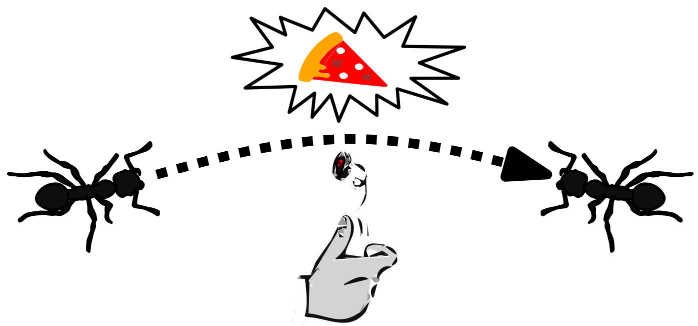
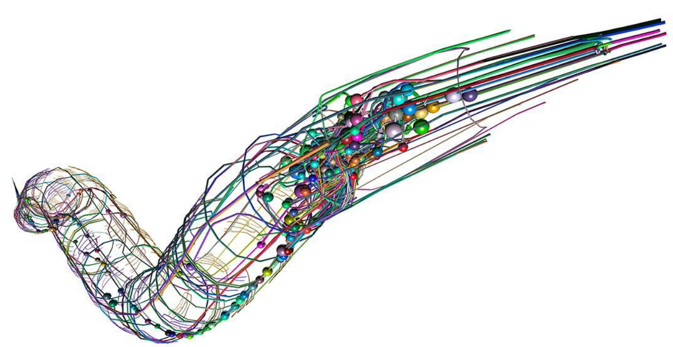


- **Neuromorphic Computing.** Theory of neural networks (algorithmic approach to theoretical neuroscience).



Thank You!