On the Metastability of Quadratic Majority Dynamics on Clustered Graphs and its Biological Implications

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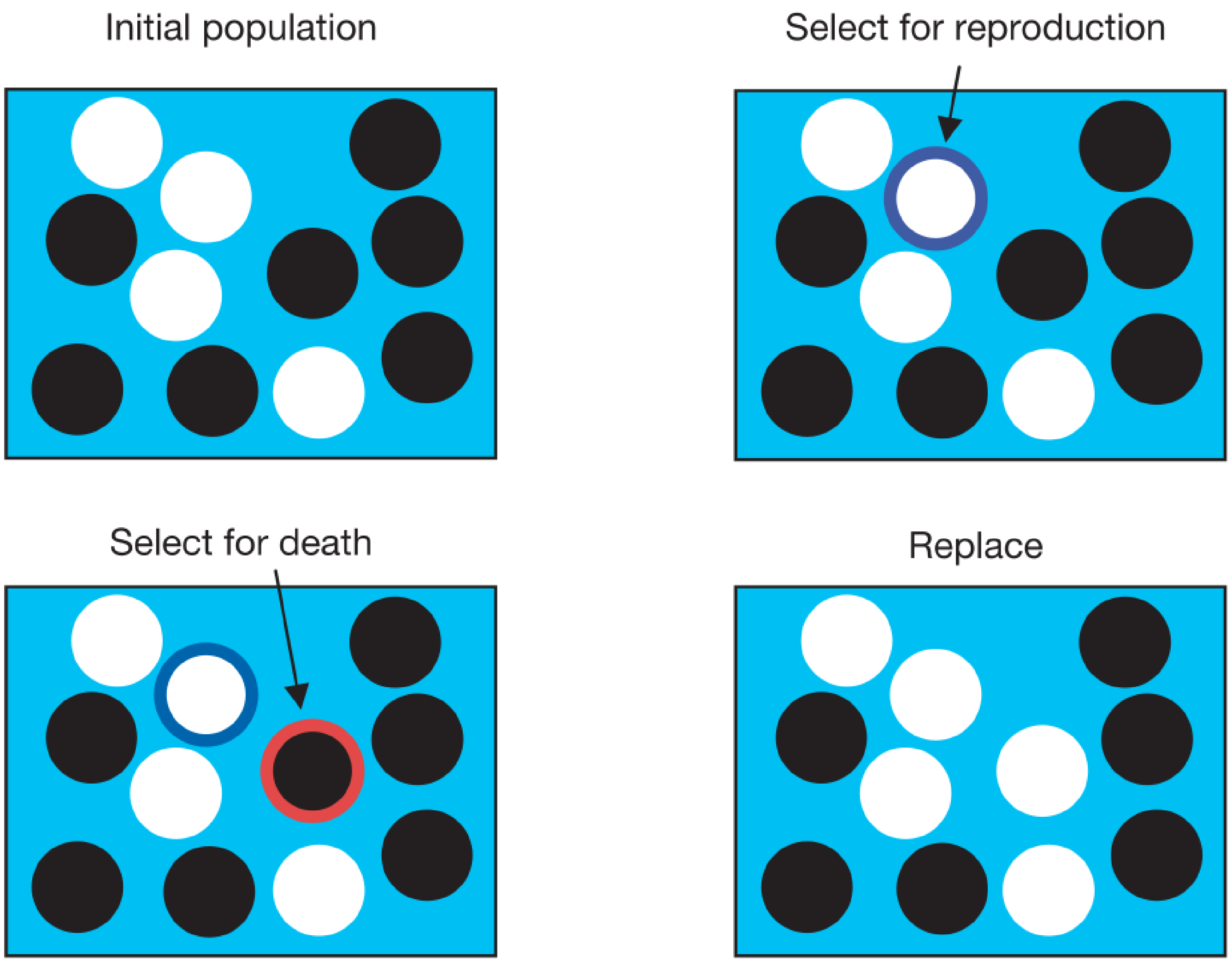
Evolutionary Dynamics on Graphs

[Lieberman, Hauert & Nowak, Nature '05]:

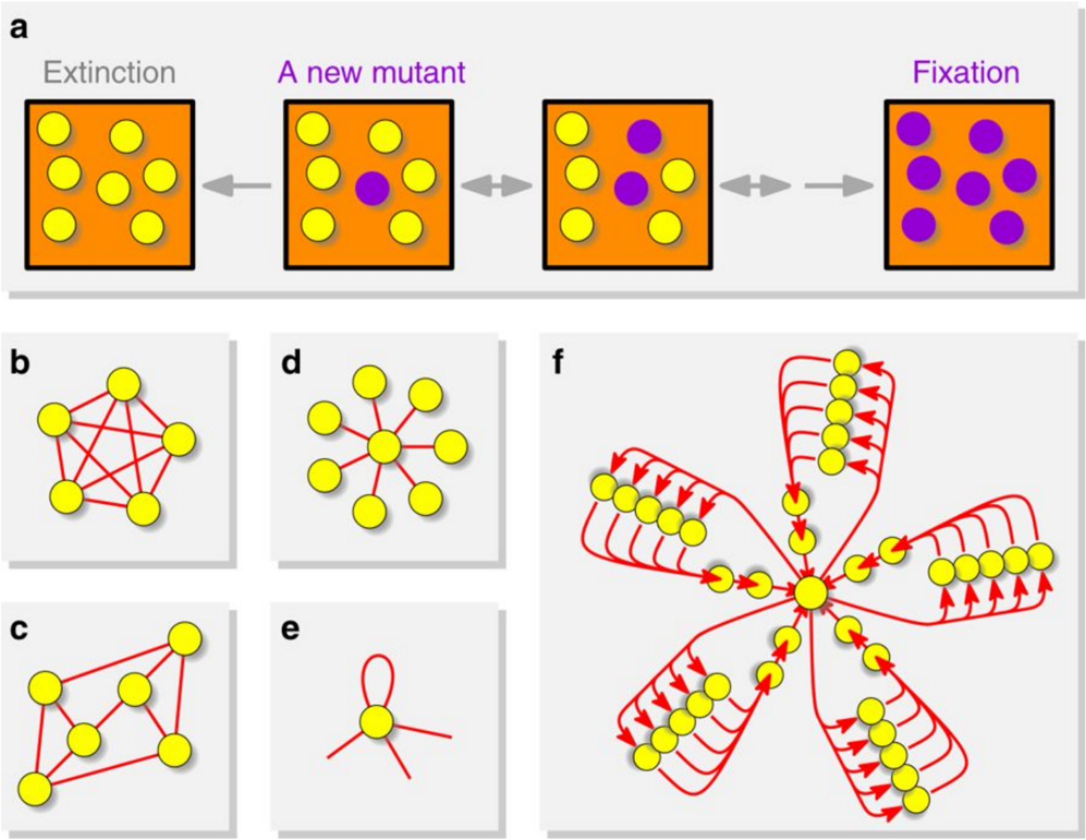


A node is selected randomly according to its fitness and it replaces a random neighbor

Picture from Lieberman et al.

The Moran Process and Fixation Probability

[Giakkoupis '16, Galanis et al. J. ACM '17, Goldberg et al.x2 '18, Pavlogiannis et al. Comm. Bio. '18]:
Probability that a mutant with fitness r conquers a population with fitness 1 on a family of graphs $\{G_n\}_n$.
Are there families G_n with probability $1 - o_n(1)$?



Picture from Pavlogiannis et al.

The Speed of Speciation

The Moran process doesn't provide an explanation for *speciation*

"What is needed now is a shift in focus to identifying more general rules and patterns in the dynamics of speciation. The crucial step in achieving this goal is the development of simple and general dynamical models that can be studied not only numerically but analytically as well. [...] Speciation is expected to be triggered by changes in the environment. Once genetic changes underlying speciation start, they go to completion very rapidly."

[Gavrilets, Evolution '03]

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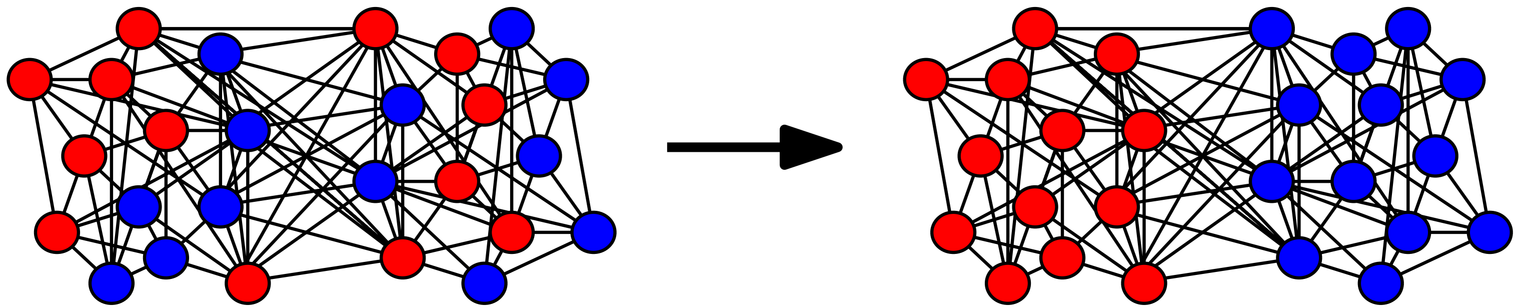
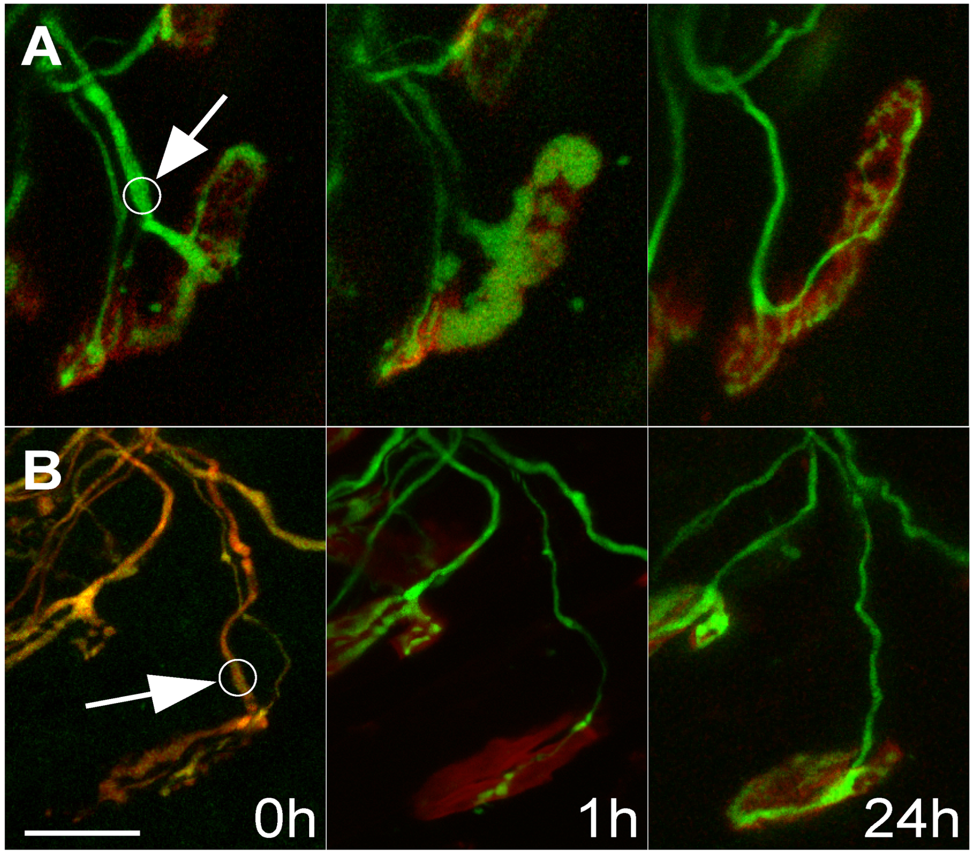
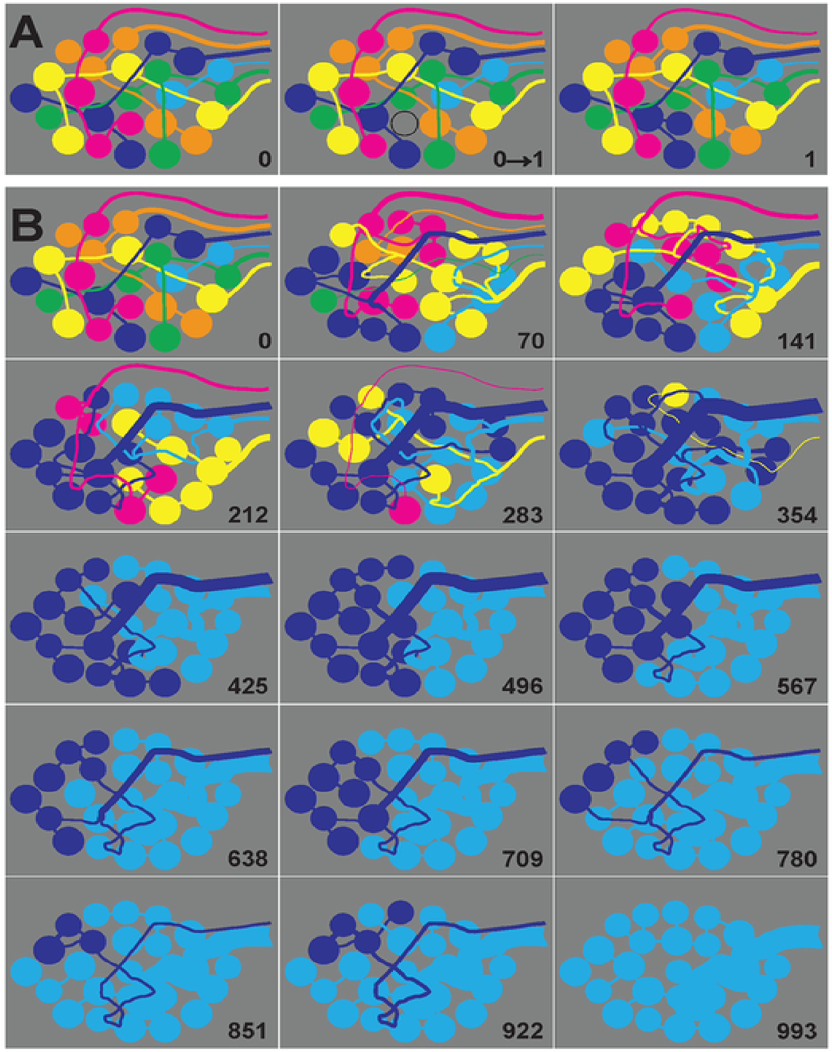
Problem: A simple evolutionary-graph-theoretic proof of principle for speciation.



Neural Innervation in Muscular Junctions

[Gan & Lichtman, Science '03; Turney & Lichtman, PLOS Bio. '12; Tapia et al., Neuron '12]:





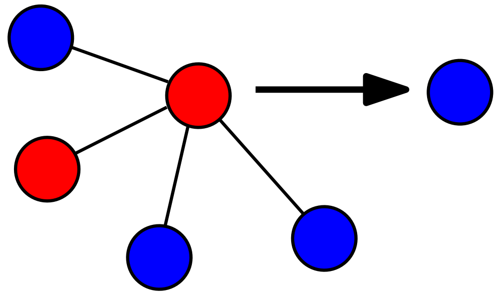
Axonal competition is modeled with a Moran process where nodes are innervation sites and colors are innervating axons.

Picture from Turney & Lichtman, PLOS Bio. '12

y-Degree Majority Dynamics

Node gets color x with probability

$$\left(\frac{\#\text{neighbors with col. } x}{\text{degree}}\right)^y$$



$y = 1 \implies$ Voter Dynamics (Moran Process)

$y = 2 \implies$ 2-Choices Dynamics

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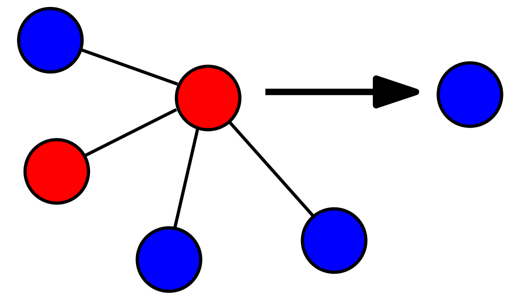

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$y = 2 \implies$ 2-Choices Dynamics

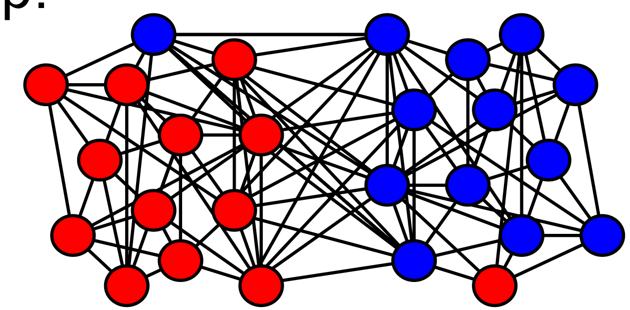
[Cooper et al.x3, ICALP'14, DISC'15, DISC'17]: 2-Choice Dynamics can be related to the *spectral structure* of the graph!

$$\sum_{x \in V} \left(\frac{B(x)}{d}\right)^2 = \|P\mathbf{1}_B\|_2^2 \le \frac{B^2}{n} + \lambda^2 B.$$

$B(x)$ blue neighbors of x, P trans. matrix of graph, $\mathbf{1}_B$ indicator vector of blue-col. nodes, B overall number of blue-col. nodes, λ second-largest eigenvalue of P

Metastability of 2-Choices Dynamics

Thm. G d-regular graph divided in 2 *clusters*, where cut is a b-regular bipartite graph. Each node initially blue or red u.a.r. If $b/d = \mathcal{O}(1/\sqrt{n})$ and spectral radius of clusters is $\mathcal{O}(n^{-\frac{1}{4}})$, then with prob. $\Omega(1)$, after $\mathcal{O}(\log n)$ *time*, clusters are *almost-monochromatic*, with *different colors*, and remains so for $n^{\Omega(1)}$ *time* w.h.p.



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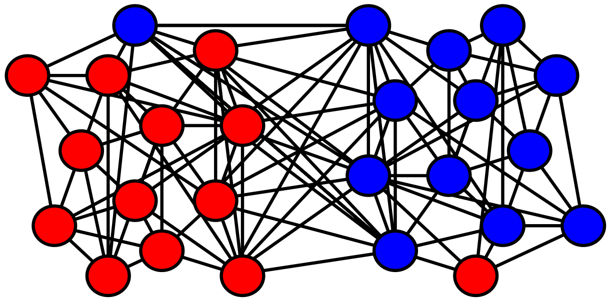

Corollaries:

Evolutionary Graph Theory. A proof of principle for speciation.

Axonal Innervation. Either the innervation sites do not exhibit spatial bottlenecks or the dynamics cannot be based on majority-like mechanism.

LPA. First analytical result on a sparse Label Propagation Algorithm (class of clustering heuristics).

Thank You!