Gossip Algorithms for Majority Consensus

Emanuele Natale
www.enatale.name

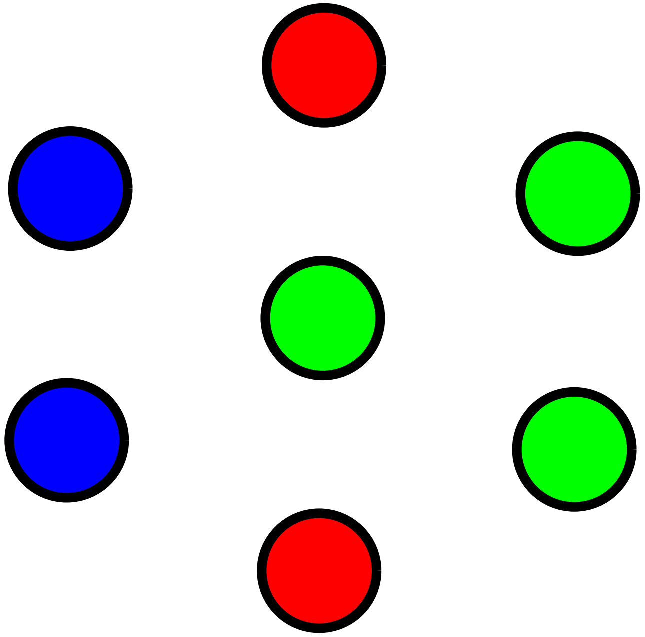
Supervisors: R. Silvestri, A. Clementi (Tor Vergata)

Research group: L. Becchetti, A. Clementi, F. Pasquale, R. Silvestri, (L. Trevisan) & me

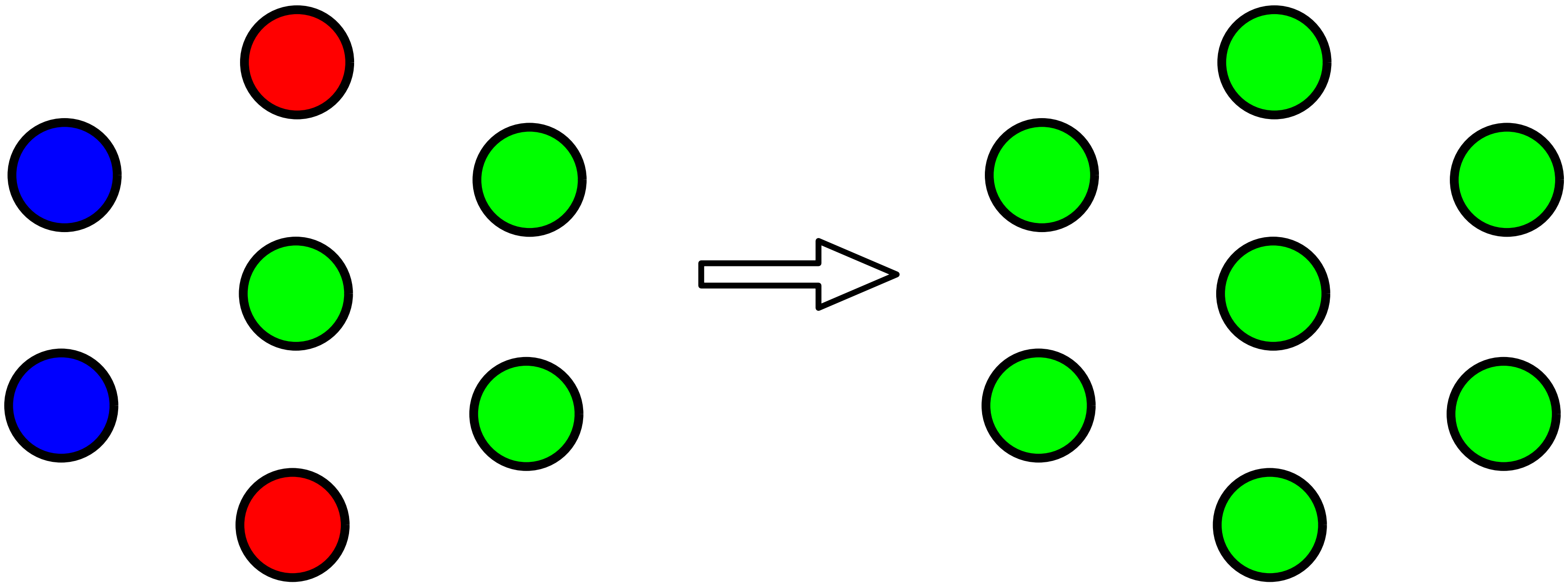
SAPIENZA
UNIVERSITÀ DI ROMA

October 12, 2015

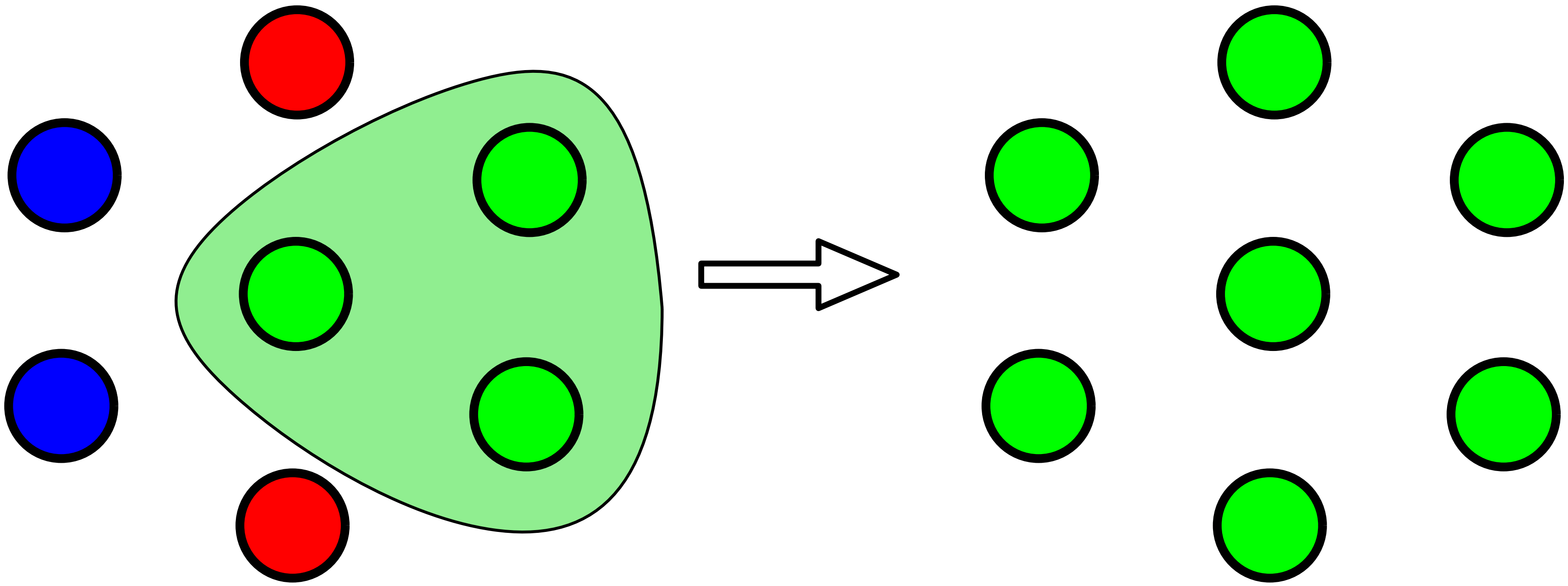
The Majority Consensus Problem



The Majority Consensus Problem



The Majority Consensus Problem



$$3 > \max\{2, 2\}$$

Gossip Algorithms

Scenario: sensor networks, peer-to-peer networks, mobile networks, vehicles networks...

$\Longrightarrow$ Distributed, unstructured, dynamical, unreliable, simple systems.

Gossip Algorithms

Scenario: sensor networks, peer-to-peer networks, mobile networks, vehicles networks...

$\Longrightarrow$ Distributed, unstructured, dynamical, unreliable, simple systems.

In a gossip algorithm the actions of each node u are

Local: uses only information from neighbors

Gossip Algorithms

Scenario: sensor networks, peer-to-peer networks, mobile networks, vehicles networks...

$\Longrightarrow$ Distributed, unstructured, dynamical, unreliable, simple systems.

In a gossip algorithm the actions of each node u are

Local: uses only information from neighbors

Lightweight: $O(\text{polylog} n)$ computation per round and $O(\text{polylog} n)$ memory

Gossip Algorithms

Scenario: sensor networks, peer-to-peer networks, mobile networks, vehicles networks...

$\Longrightarrow$ Distributed, unstructured, dynamical, unreliable, simple systems.

In a gossip algorithm the actions of each node u are

Local: uses only information from neighbors

Lightweight: $O(\text{polylog} n)$ computation per round and $O(\text{polylog} n)$ memory

Asynchronous: no global clock

Gossip Algorithms

Scenario: sensor networks, peer-to-peer networks, mobile networks, vehicles networks...

$\Longrightarrow$ Distributed, unstructured, dynamical, unreliable, simple systems.

In a gossip algorithm the actions of each node u are

- Local: uses only information from neighbors
- Lightweight: $O(\text{polylog} n)$ computation per round and $O(\text{polylog} n)$ memory
- Asynchronous: no global clock
- Robust: it is not affected by "small" changes in the network

Gossip Algorithms

Scenario: sensor networks, peer-to-peer networks, mobile networks, vehicles networks...

$\Longrightarrow$ Distributed, unstructured, dynamical, unreliable, simple systems.

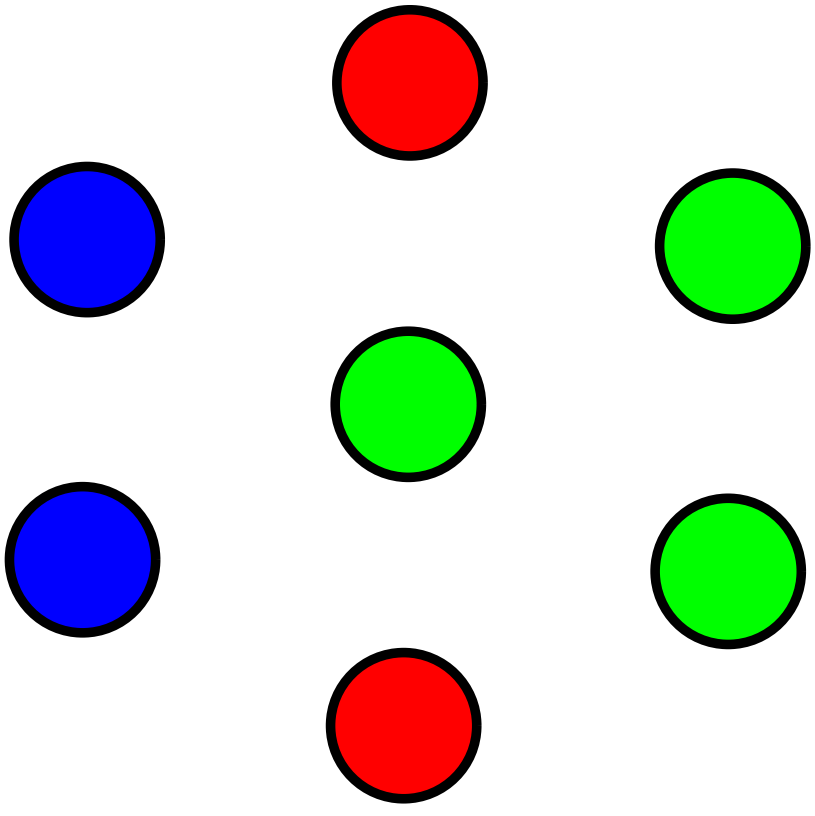
In a gossip algorithm the actions of each node u are

- Local: uses only information from neighbors
- Lightweight: $O(\text{polylog}n)$ computation per round and $O(\text{polylog}n)$ memory
- Asynchronous: no global clock
- Robust: it is not affected by "small" changes in the network

Theoretical interest:
what can we do with minimal assumptions?

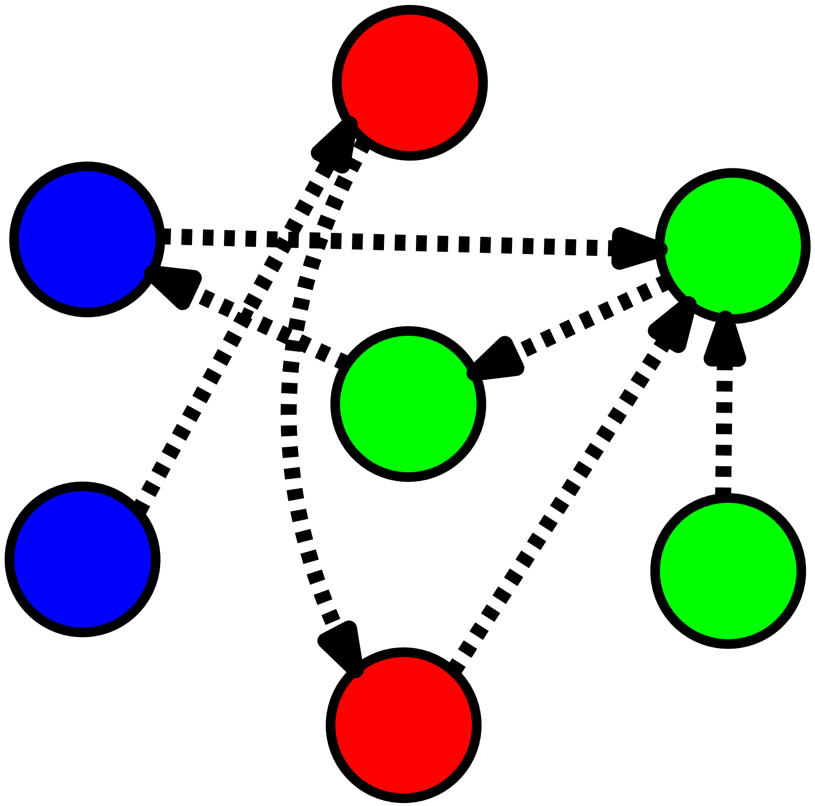
Uniform Gossip Model

At each round, each node can communicate with a finite number of nodes, chosen uniformly at random



Uniform Gossip Model

At each round, each node can communicate with a finite number of nodes, chosen uniformly at random



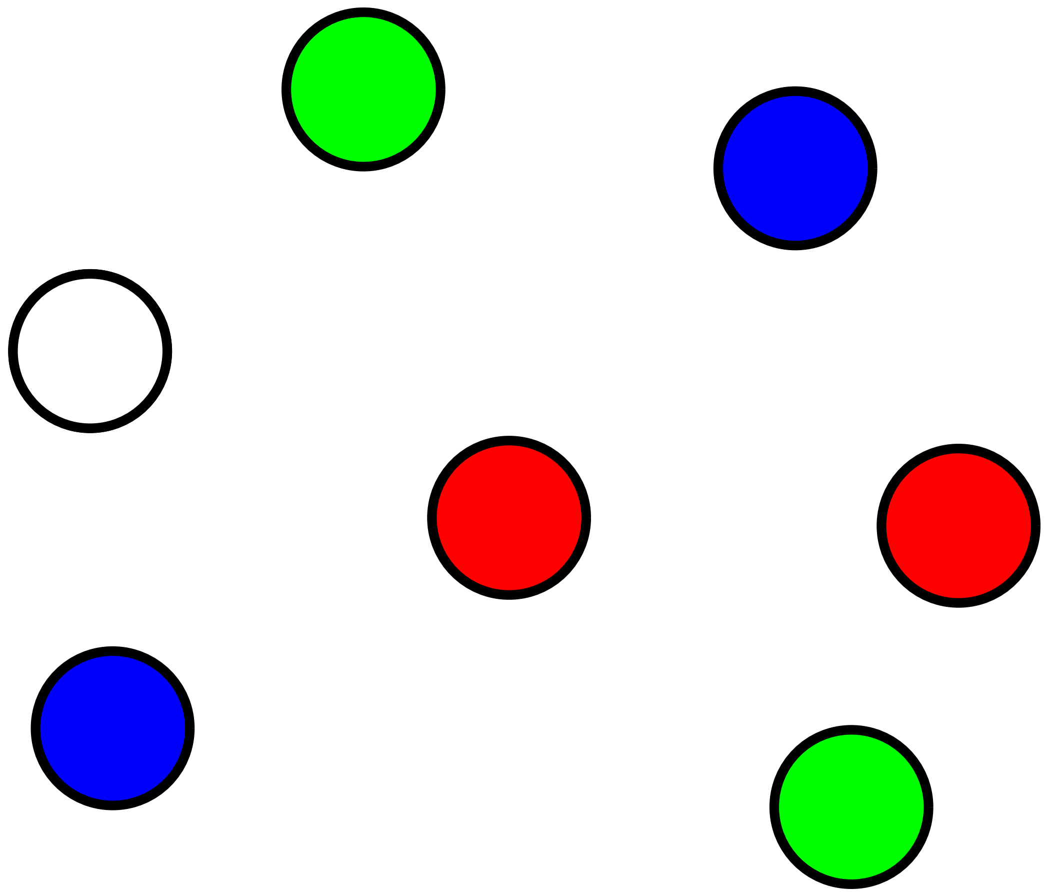
(Some) Related Works

	Mem. & mess. size	**# of colors**	**Time efficiency**	**Comm. Model**
Kempe et al. FOCS '03	$O(k \log n)$	any	$O(\log n)$	$\mathcal{GOSSIP}$
Angluin et al. DISC '07 Perron et al. INFOCOM '09	$\Theta(1)$	2	$O(\log n)$	Sequential
Doerr et al. SPAA '11	$\Theta(1)$	2	$O(\log n)$	$\mathcal{GOSSIP}$
Babaee et al. Comp. J. '12 Jung et al. ISIT '12	$O(\log k)$	Constant	$O(\log n)$	Sequential

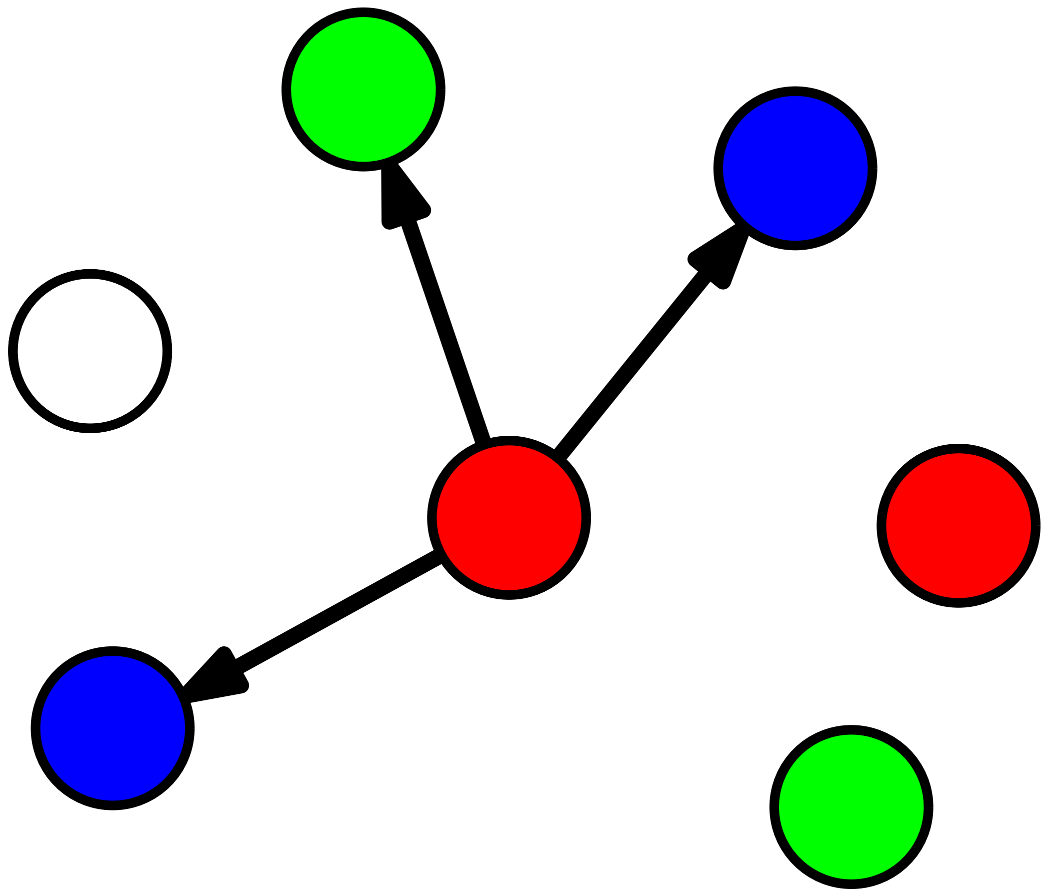
(Some) Related Works

	Mem. & mess. size	# of colors	Time efficiency	Comm. Model
Kempe et al. FOCS '03	$O(k \log n)$ ✗	any ✓	$O(\log n)$ ✓	$\mathcal{GOSSIP}$ ✓
Angluin et al. DISC '07 Perron et al. INFOCOM '09	$\Theta(1)$ ✓	2 ✗	$O(\log n)$ ✓	Sequential ✗
Doerr et al. SPAA '11	$\Theta(1)$ ✓	2 ✗	$O(\log n)$ ✓	$\mathcal{GOSSIP}$ ✓
Babaee et al. Comp. J. '12 Jung et al. ISIT '12	$O(\log k)$ ✓	Constant ✗	$O(\log n)$ ✓	Sequential ✗

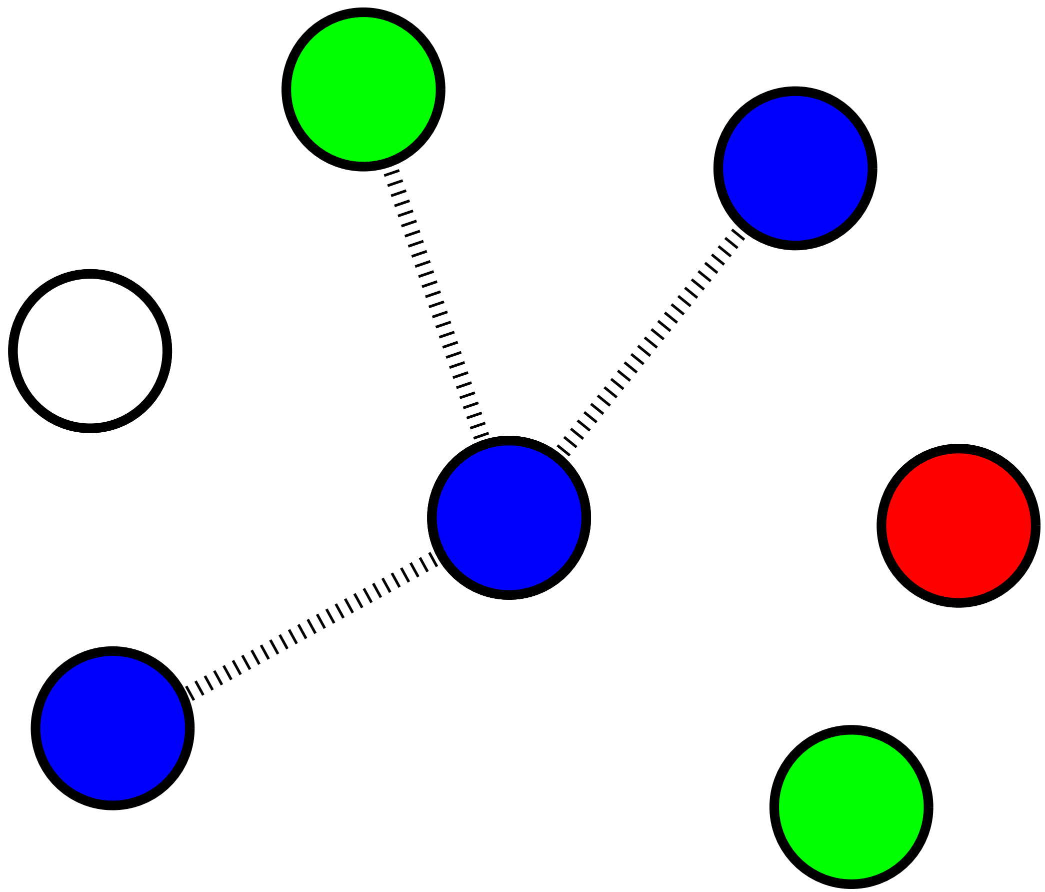
The 3-Majority Protocol



The 3-Majority Protocol

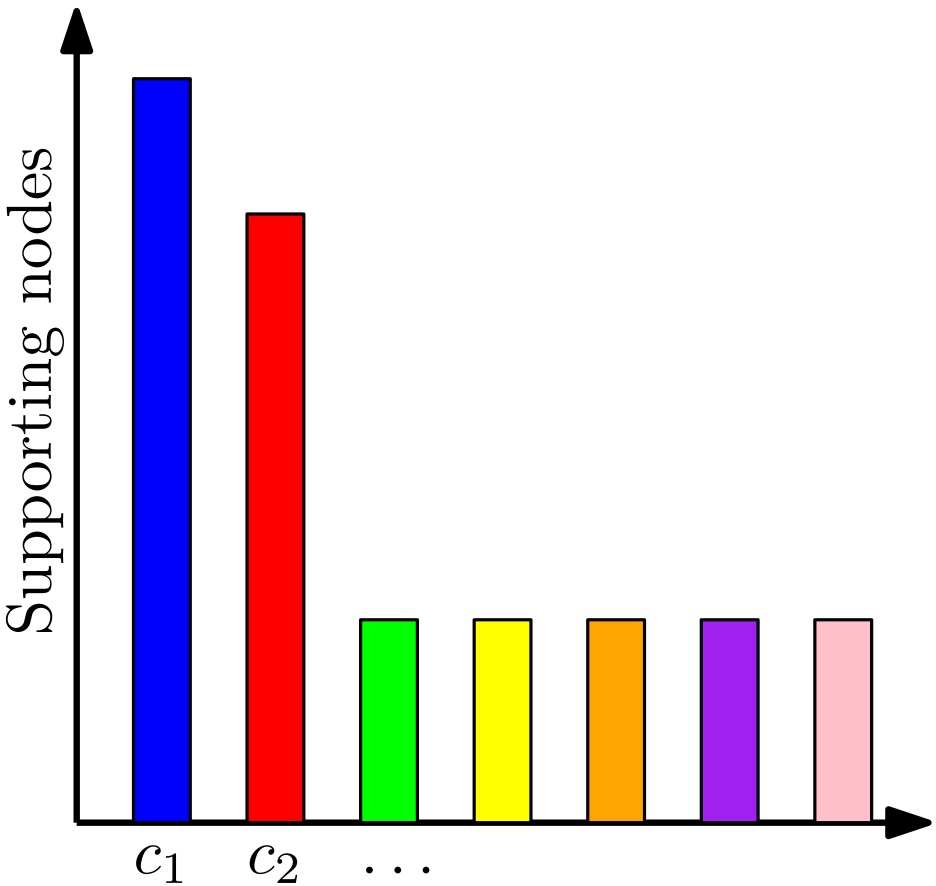


The 3-Majority Protocol

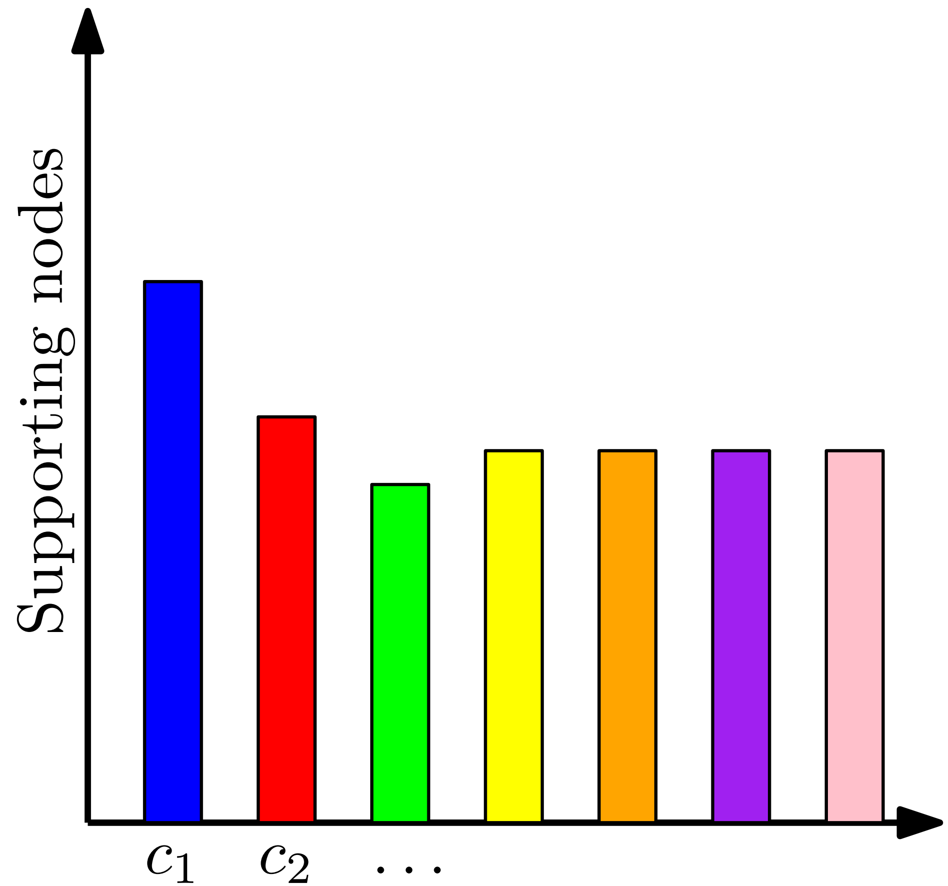


Key Parameter of 3-Majority

$c_i^{(t)} := |\{i\text{-colored nodes}\}|$



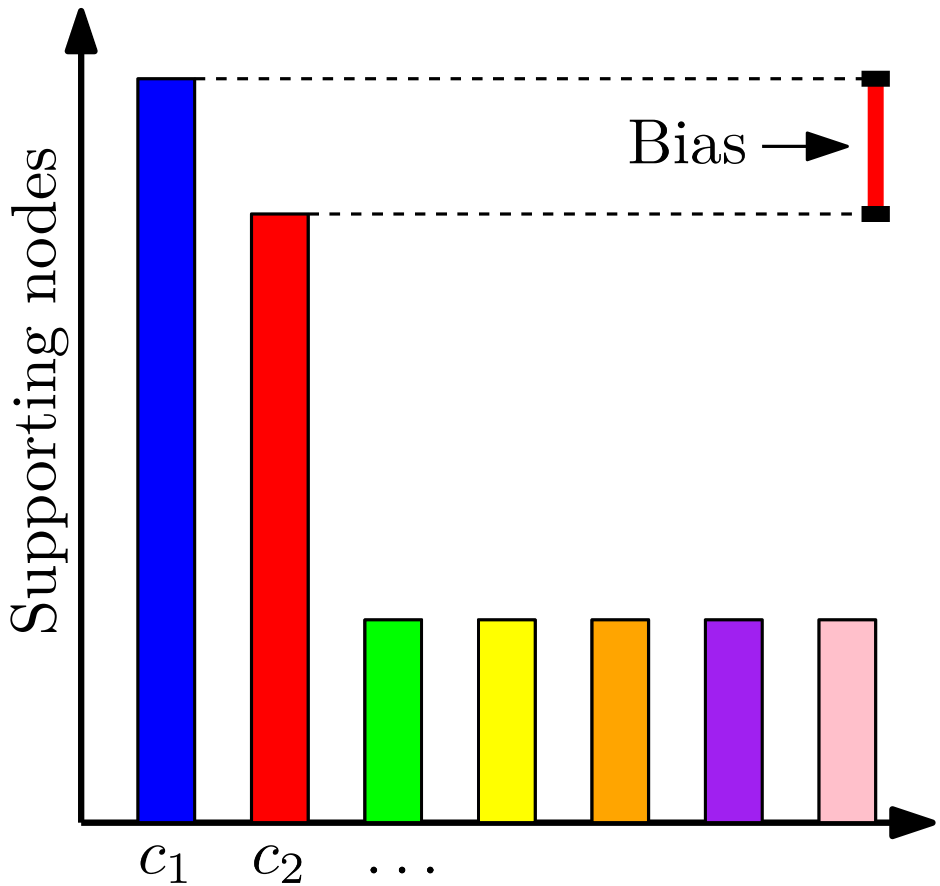
color 1 is the plurality

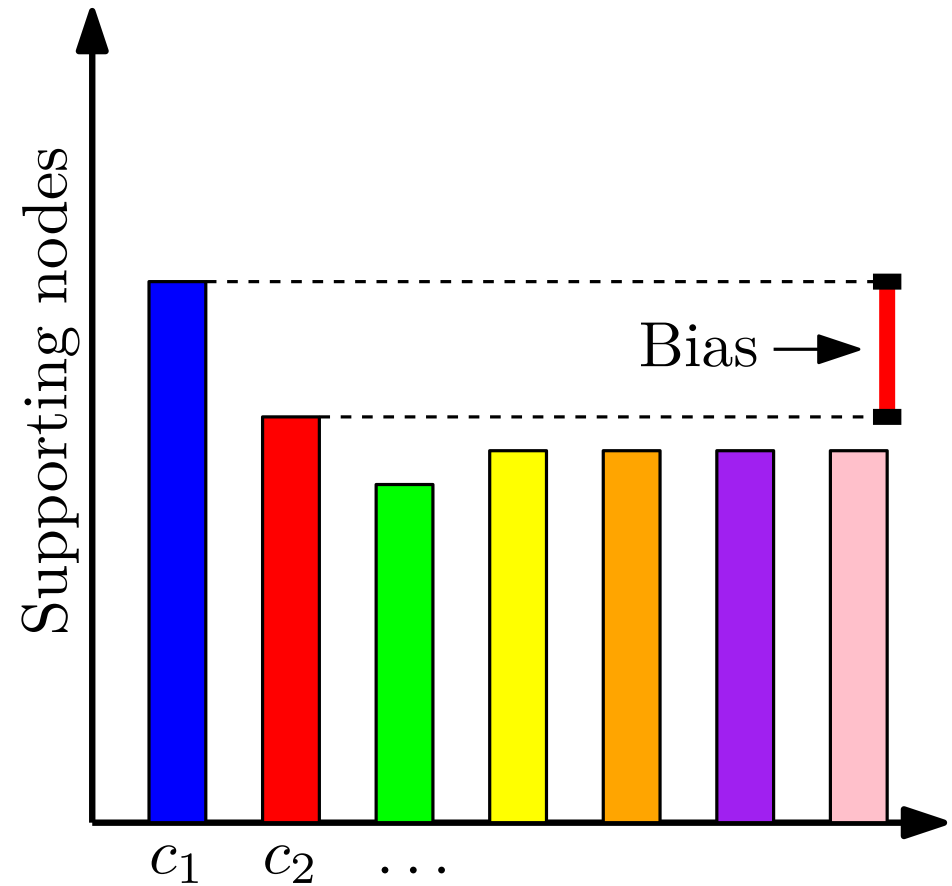


Key Parameter of 3-Majority

$c_i^{(t)} := |\{i\text{-colored nodes}\}|$

color 1 is the plurality





Convergence of 3-Majority [SPAA '14]

Thm. From any configuration with $k < \sqrt[3]{n}$ colors, such that

$$s \geq 22\sqrt{2kn\log n},$$

the 3-majority protocol converges to the majority opinion in $O(2k\log n)$ rounds w.h.p., even in the presence of a $O(\sqrt{n})$-bounded dynamic adversary.

Convergence of 3-Majority [SPAA '14]

Thm. From any configuration with $k < \sqrt[3]{n}$ colors, such that

$$s \geq 22\sqrt{2kn\log n},$$

the 3-majority protocol converges to the majority opinion in $O(2k\log n)$ rounds w.h.p., even in the presence of a $O(\sqrt{n})$-bounded dynamic adversary.

	Mem. & mess. size	**# of colors**	**Time efficiency**	**Comm. Model**
SPAA '14	$O(\log k)$	$n^{\Theta(1)}$	$O(k\log n)$	$\mathcal{GOSSIP}$

Convergence of 3-Majority [SPAA '14]

Thm. From any configuration with $k < \sqrt[3]{n}$ colors, such that

$$s \geq 22\sqrt{2kn\log n},$$

the 3-majority protocol converges to the majority opinion in $O(2k\log n)$ rounds w.h.p., even in the presence of a $O(\sqrt{n})$-bounded dynamic adversary.

	Mem. & mess. size	**# of colors**	**Time efficiency**	**Comm. Model**
SPAA '14	$O(\log k)$ ✓	$n^{\Theta(1)}$ ✓	$O(k\log n)$	$\mathcal{GOSSIP}$ ✓

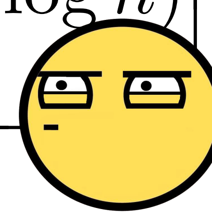
Convergence of 3-Majority [SPAA '14]

Thm. From any configuration with $k < \sqrt[3]{n}$ colors, such that

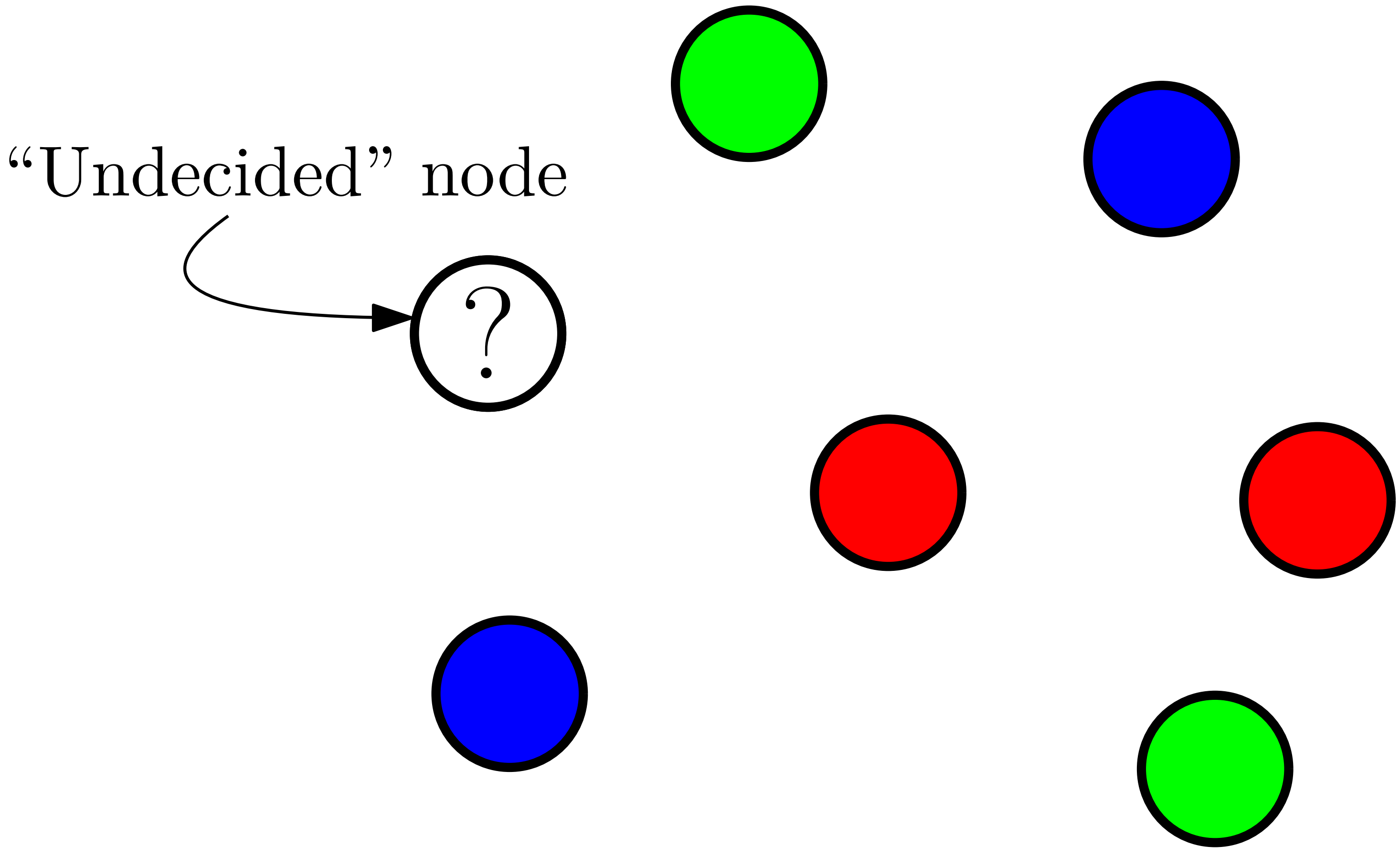
$$s \geq 22\sqrt{2kn\log n},$$

the 3-majority protocol converges to the majority opinion in $O(2k\log n)$ rounds w.h.p., even in the presence of a $O(\sqrt{n})$-bounded dynamic adversary.

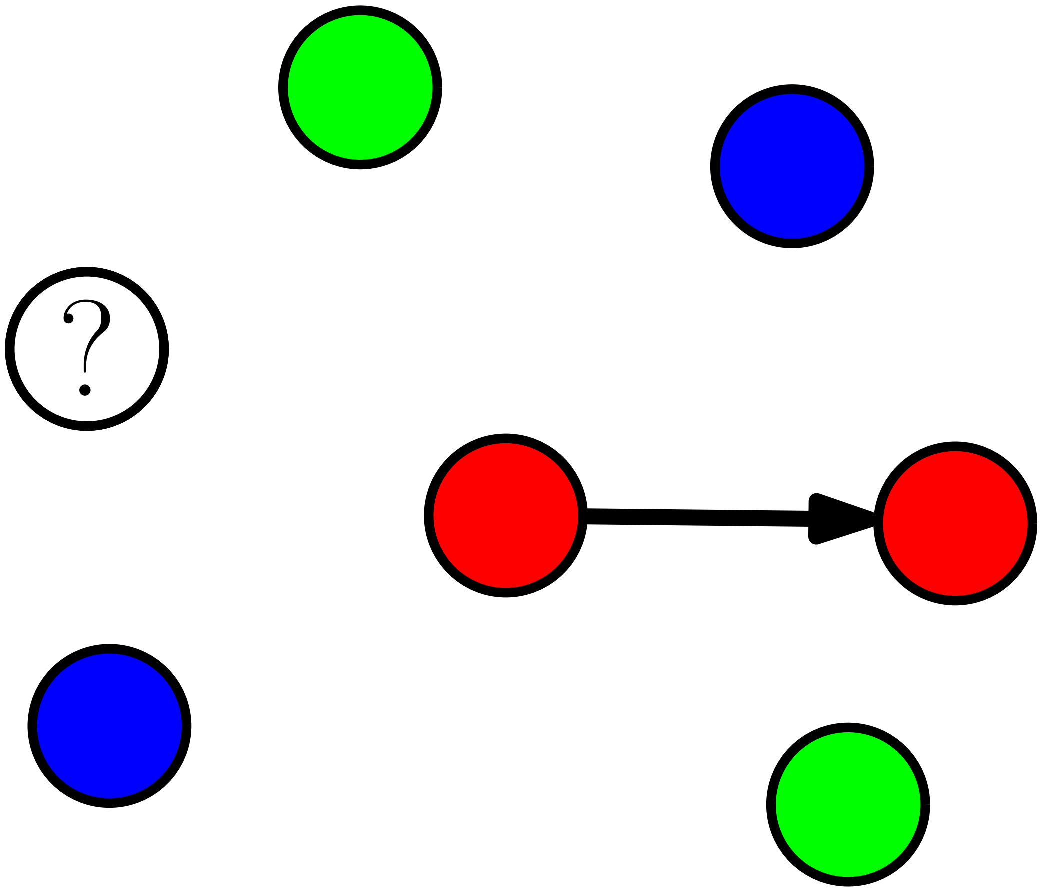
	Mem. & mess. size	**# of colors**	**Time efficiency**	**Comm. Model**
SPAA '14	$O(\log k)$	$n^{\Theta(1)}$	$O(k\log n)$	$\mathcal{GOSSIP}$



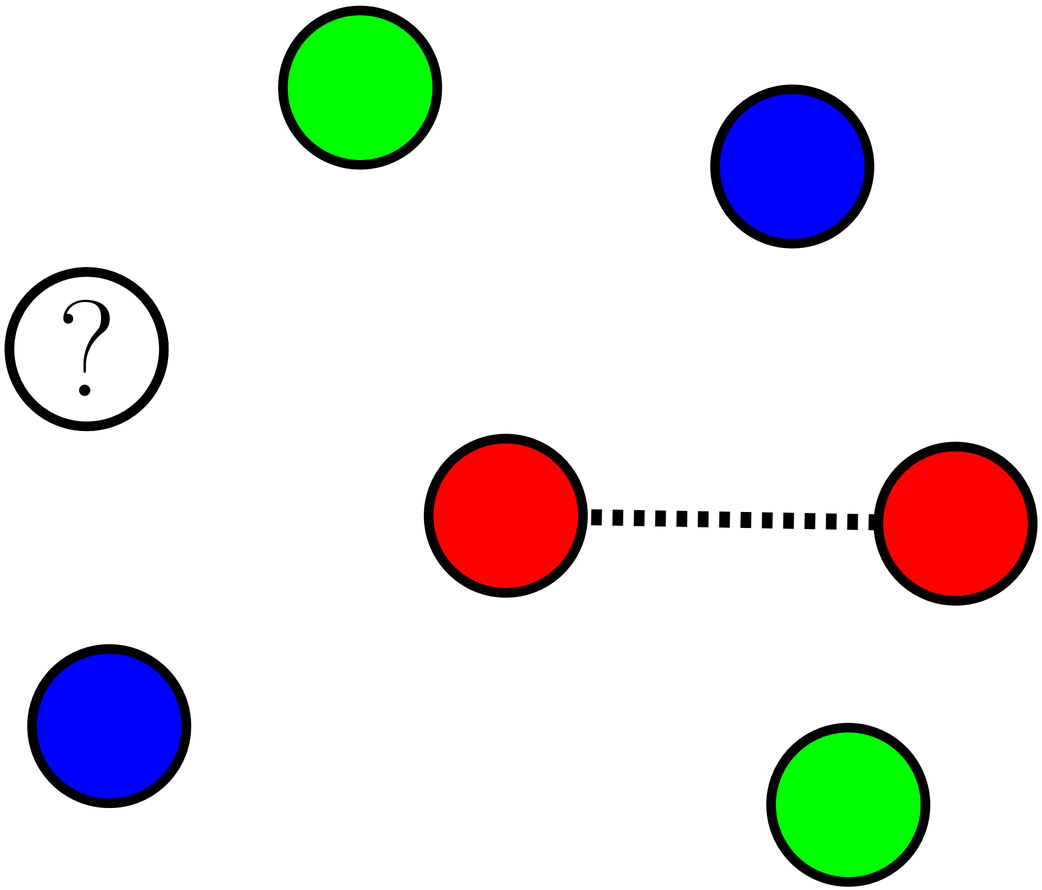
The Undecided-State Protocol



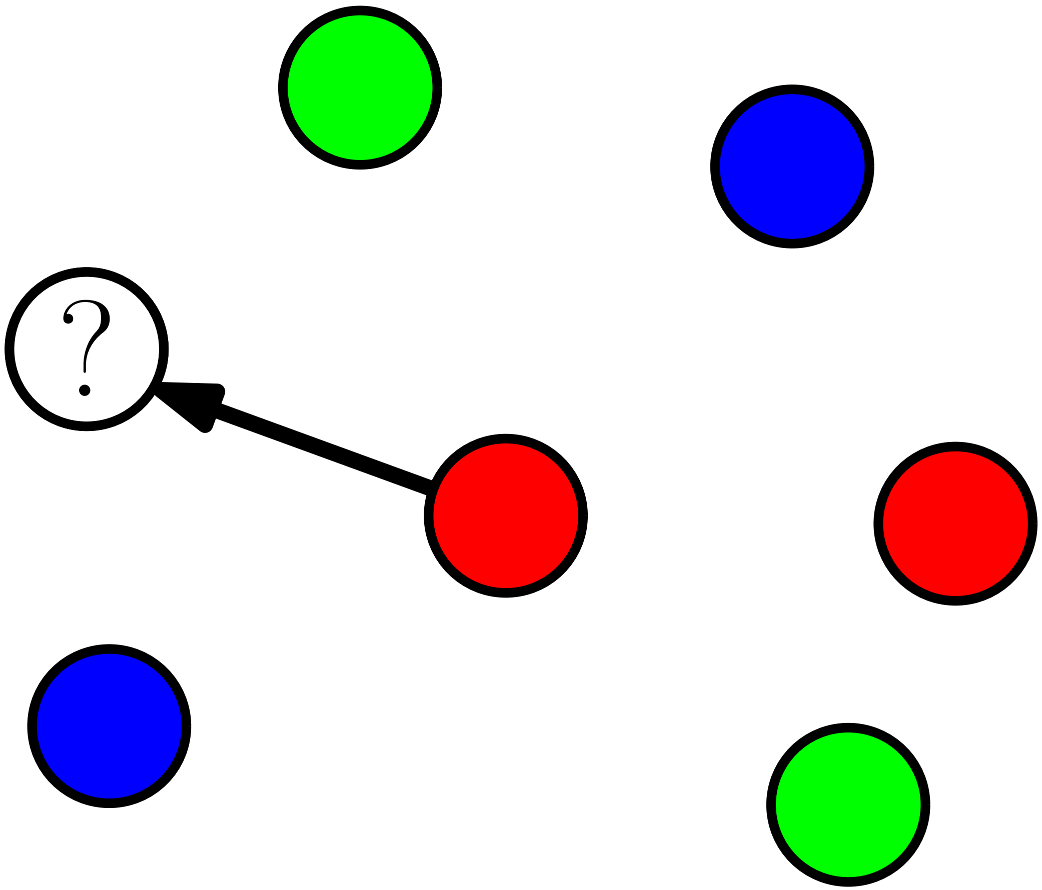
The Undecided-State Protocol



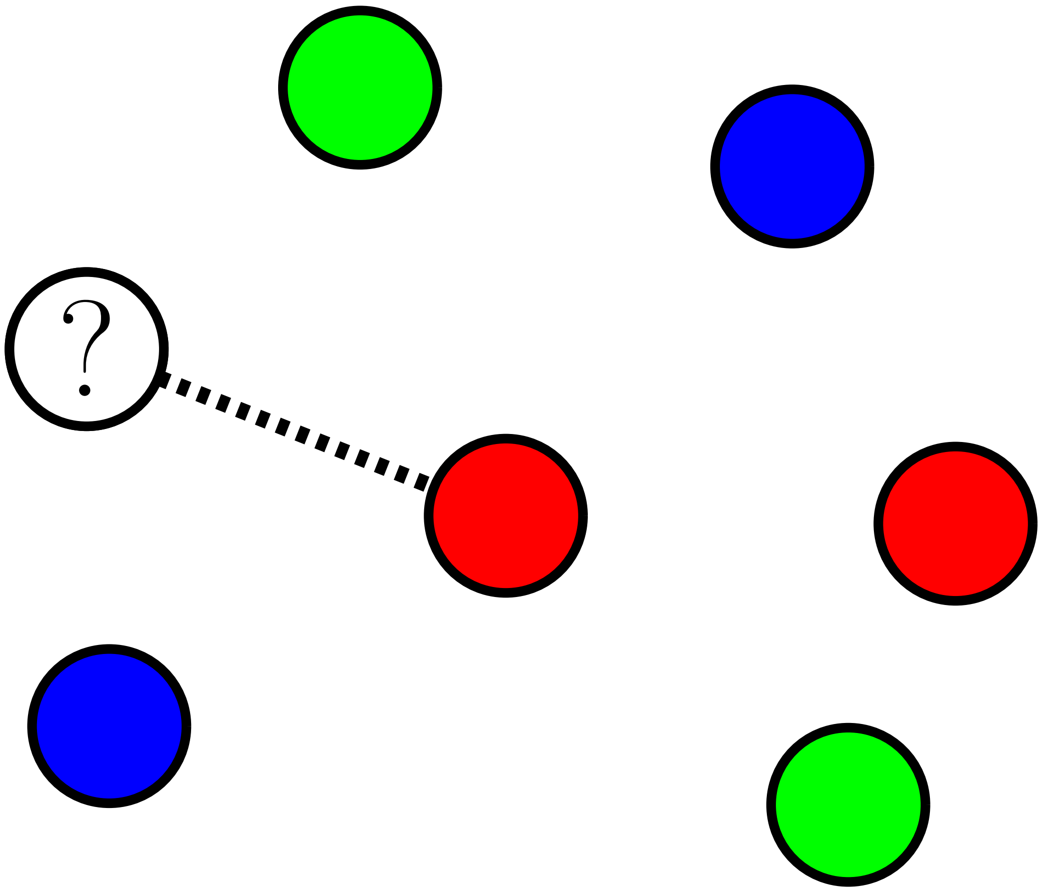
The Undecided-State Protocol



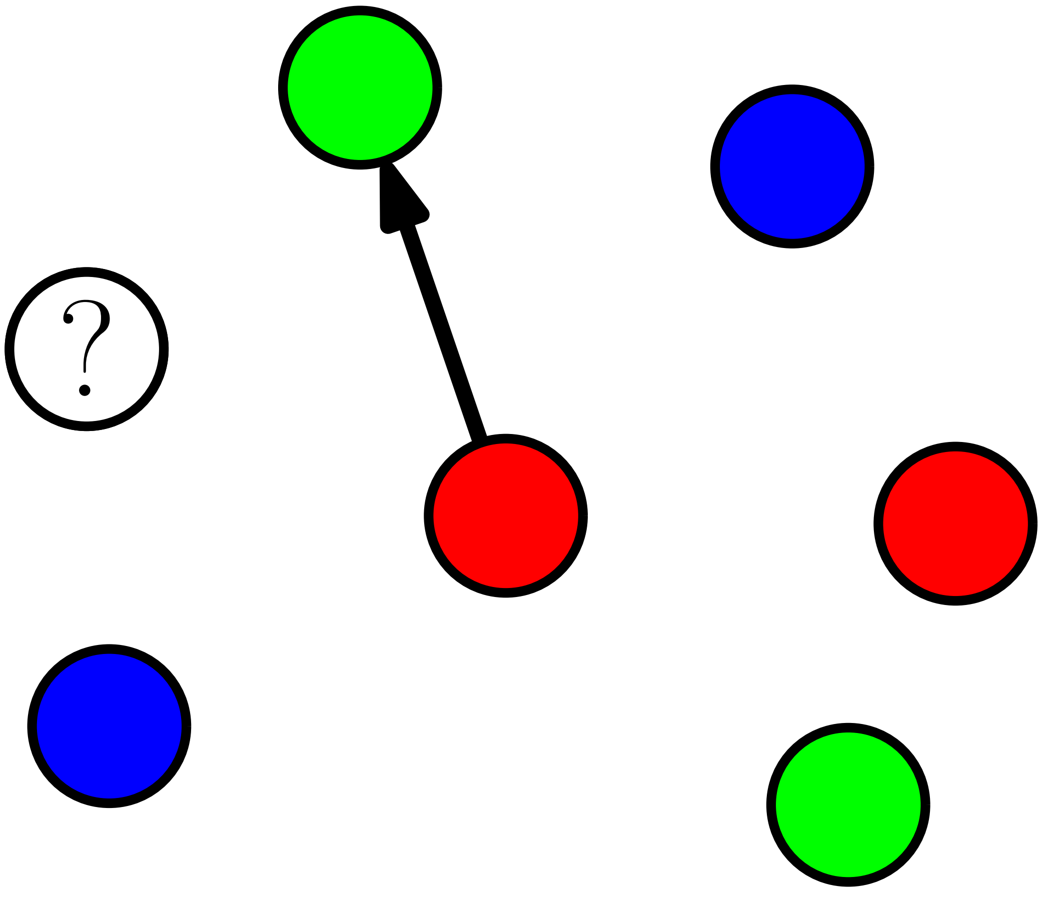
The Undecided-State Protocol



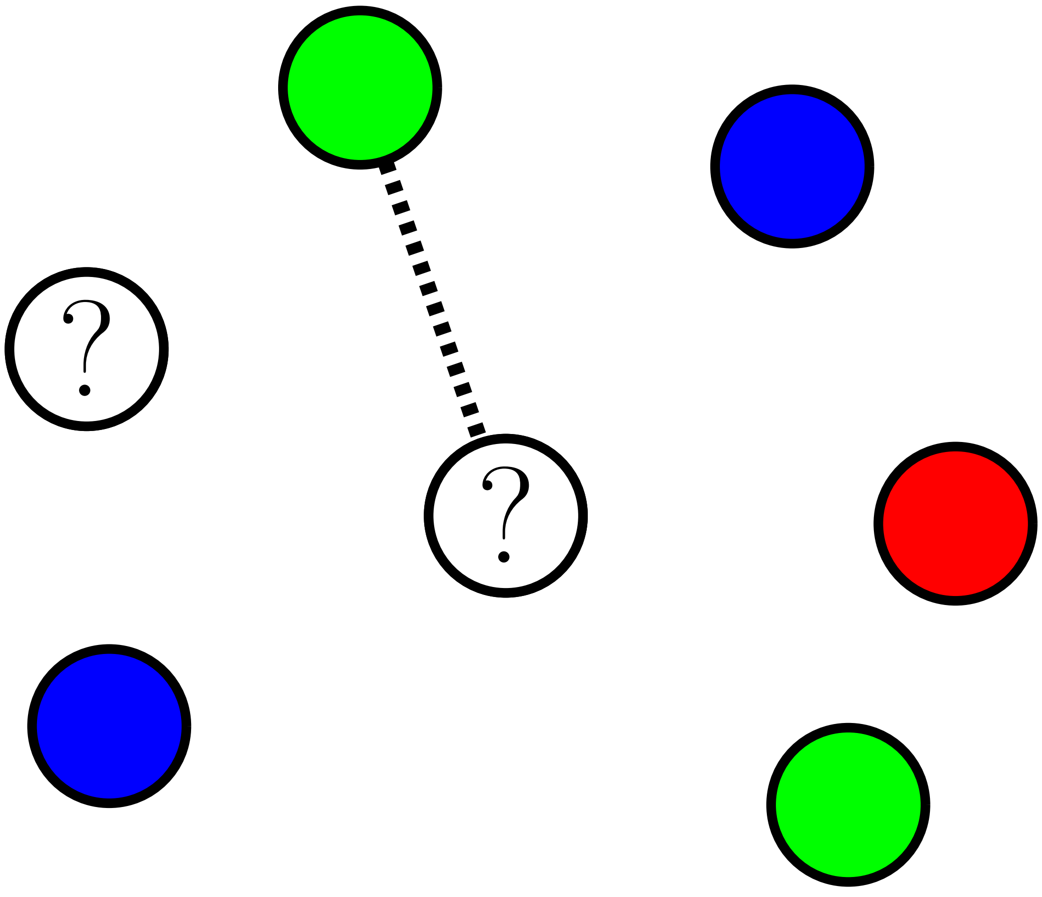
The Undecided-State Protocol



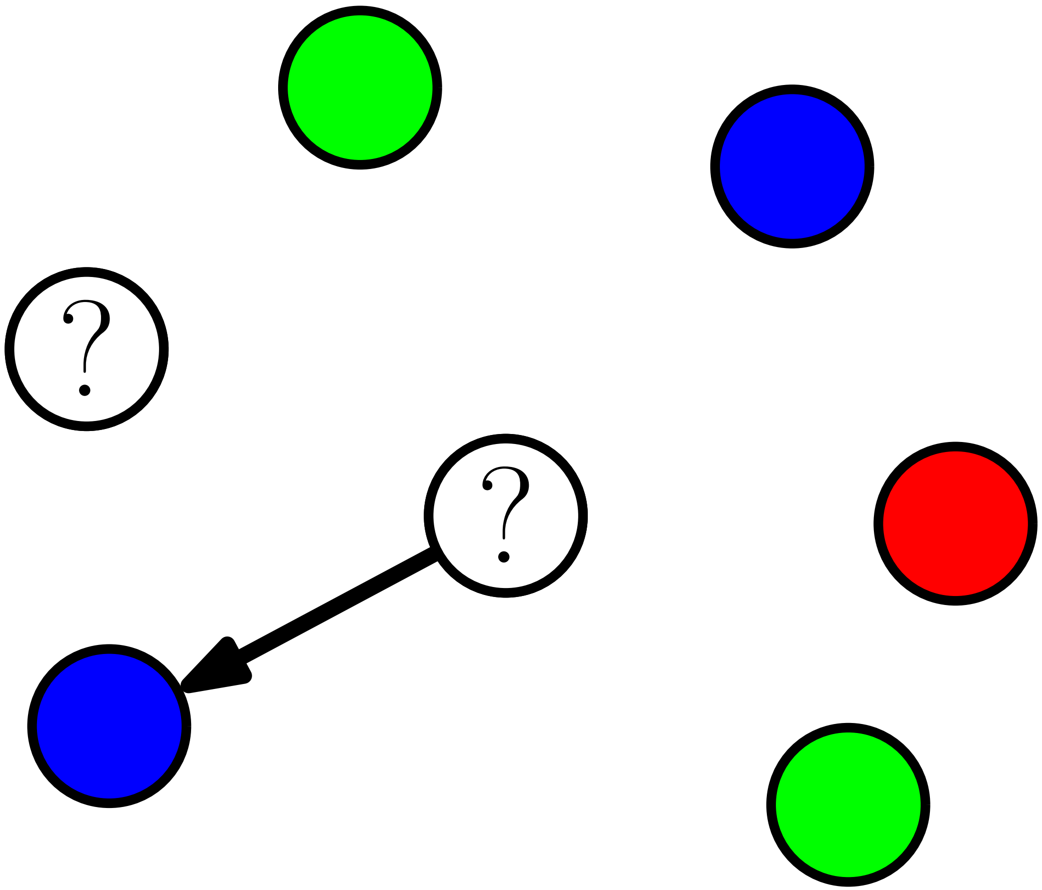
The Undecided-State Protocol



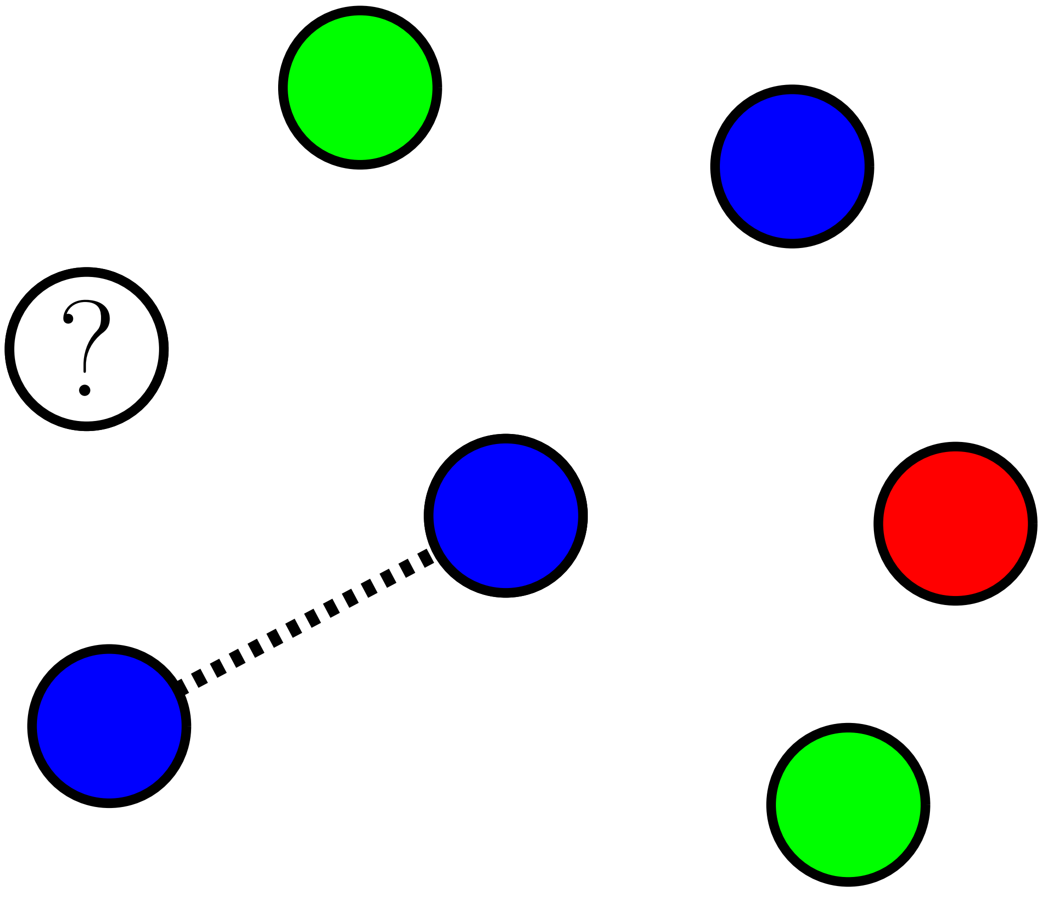
The Undecided-State Protocol



The Undecided-State Protocol



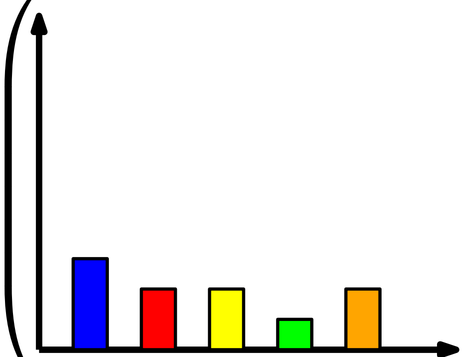
The Undecided-State Protocol



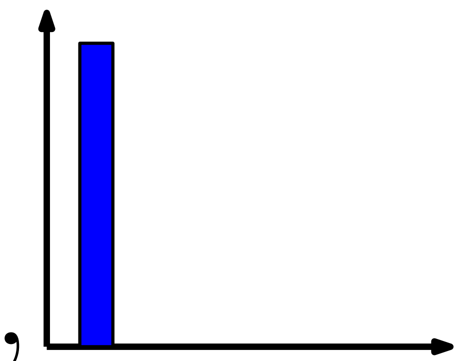
The Monochromatic Distance

$c_i^{(t)} :=$ # nodes with color i, $\quad \mathbf{c}^{(t)} :=$ configuration at time t.

$$\mathrm{md}(\mathbf{c}^{(\mathbf{0})}) := \sum_{\mathrm{i}=1}^{\mathrm{k}} \left(\frac{\mathrm{c}_{\mathrm{i}}^{(0)}}{\mathrm{c}_{1}^{(0)}} \right)^2 = 1 + \mathcal{D} \Big($$



$,$

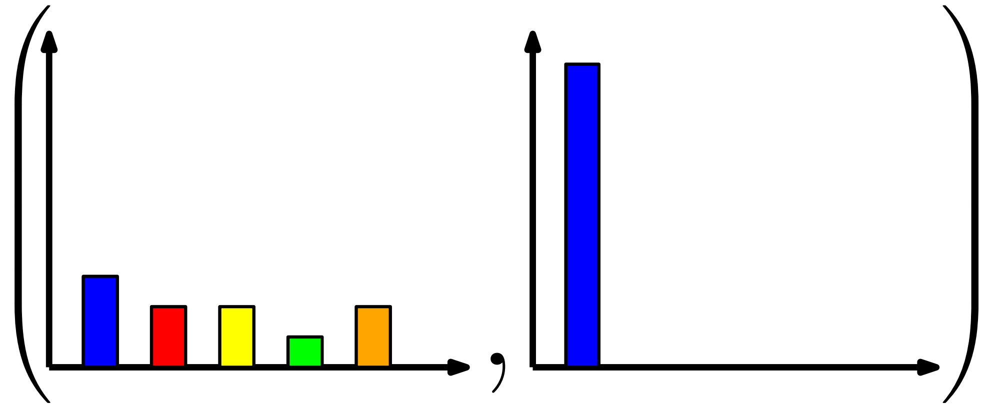


$$\Big)$$

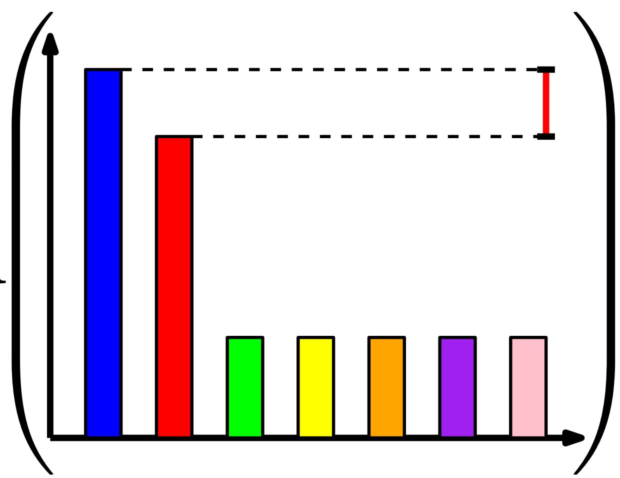
The Monochromatic Distance

$c_i^{(t)} :=$ # nodes with color i, $\quad \mathbf{c}^{(t)} :=$ configuration at time t.

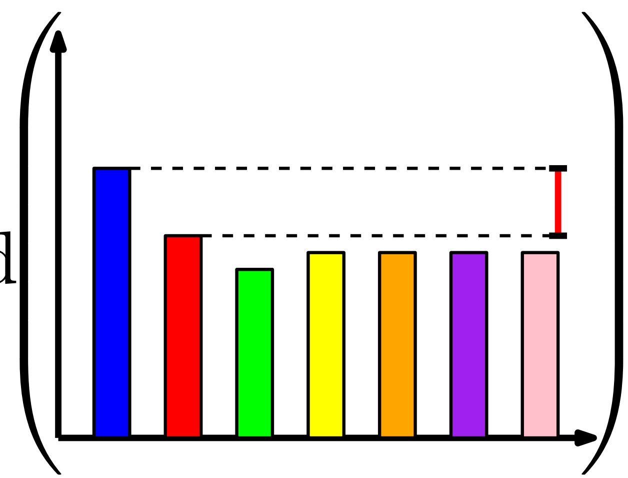
$$\mathrm{md}(\mathbf{c}^{(\mathbf{0})}) := \sum_{\mathrm{i}=1}^{\mathrm{k}} \left(\frac{\mathrm{c}_{\mathrm{i}}^{(0)}}{\mathrm{c}_{1}^{(0)}}\right)^2 = 1 + \mathcal{D}$$



$1 \le \mathrm{md}$



$\ll \mathrm{md}$



$\le k$

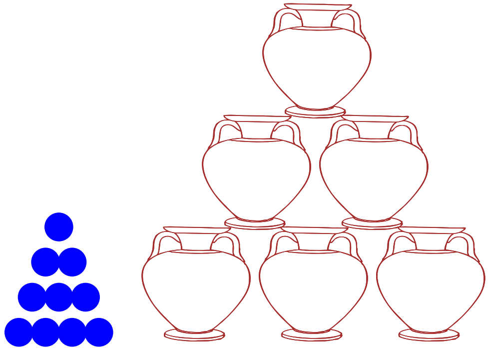
Convergence of the Undecided-State [SODA '15]

Theorem
If $k = O\left((n/\log n)^{1/3}\right)$ and $c_1 \geq (1+\epsilon) \cdot c_2$, then w.h.p. the Undecided-State Dynamics reaches plurality consensus in $O\left(\mathrm{md}(\mathbf{c}^{(0)}) \cdot \log \mathrm{n}\right)$ rounds.

Thank You!

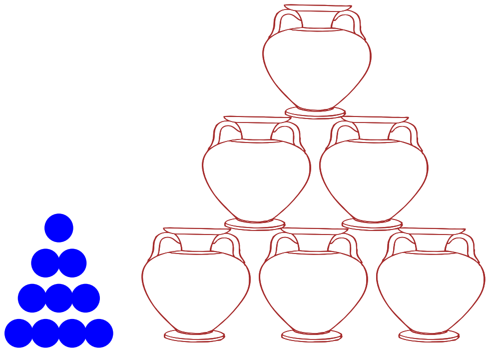
Other Stuff with My Group

Probabilistic Self-Stabilization:
Repeated Balls into Bins [SPAA '15]



Other Stuff with My Group

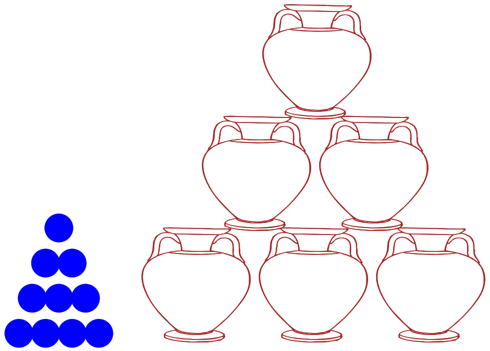
Probabilistic Self-Stabilization:
Repeated Balls into Bins [SPAA '15]



Valid Almost-Consensus against
Dynamic Adversaries [SODA '16]

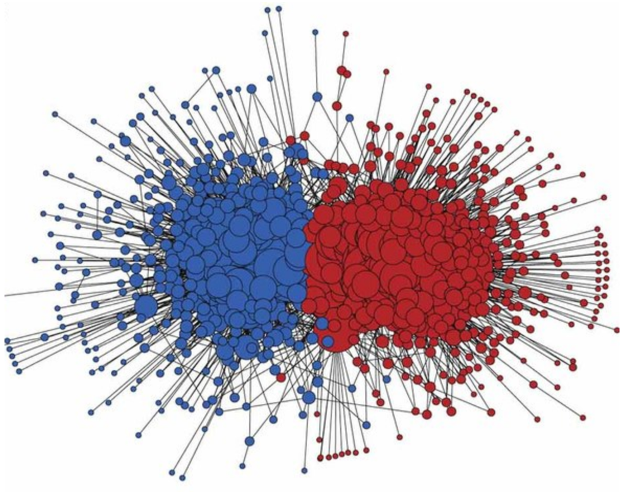
Other Stuff with My Group

Probabilistic Self-Stabilization:
Repeated Balls into Bins [SPAA '15]



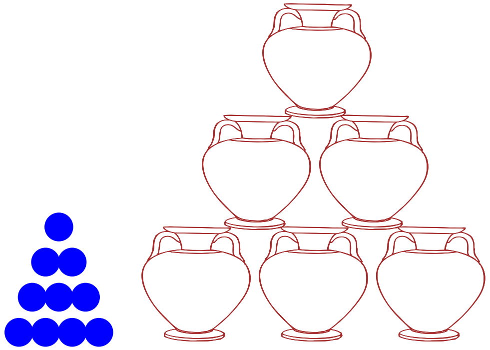
Valid Almost-Consensus against
Dynamic Adversaries [SODA '16]

Distributed Community Detection
in Stochastic Block Models
[TCS '15 + Coming soon]

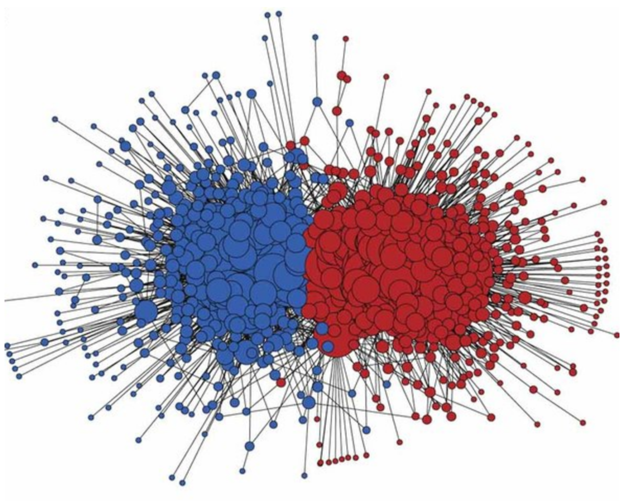


Other Stuff with My Group

Probabilistic Self-Stabilization:
Repeated Balls into Bins [SPAA '15]



Valid Almost-Consensus against
Dynamic Adversaries [SODA '16]



Distributed Community Detection
in Stochastic Block Models
[TCS '15 + Coming soon]

SIMONS INSTITUTE for the Theory of Computing

Jan-May at the Simons Institute

Other Stuff with Others

With L. Gualà and S. Leucci:
NP-Hardness of Match-3 Games
(Candy Crush) [CIG '14]

Other Stuff with Others

With L. Gualà and S. Leucci:
NP-Hardness of Match-3 Games
(Candy Crush) [CIG '14]

With D. Kaaser and F. Mallmann-Trenn:
Deterministic Binary Majority [brief ann. in DISC '15]

Other Stuff with Others

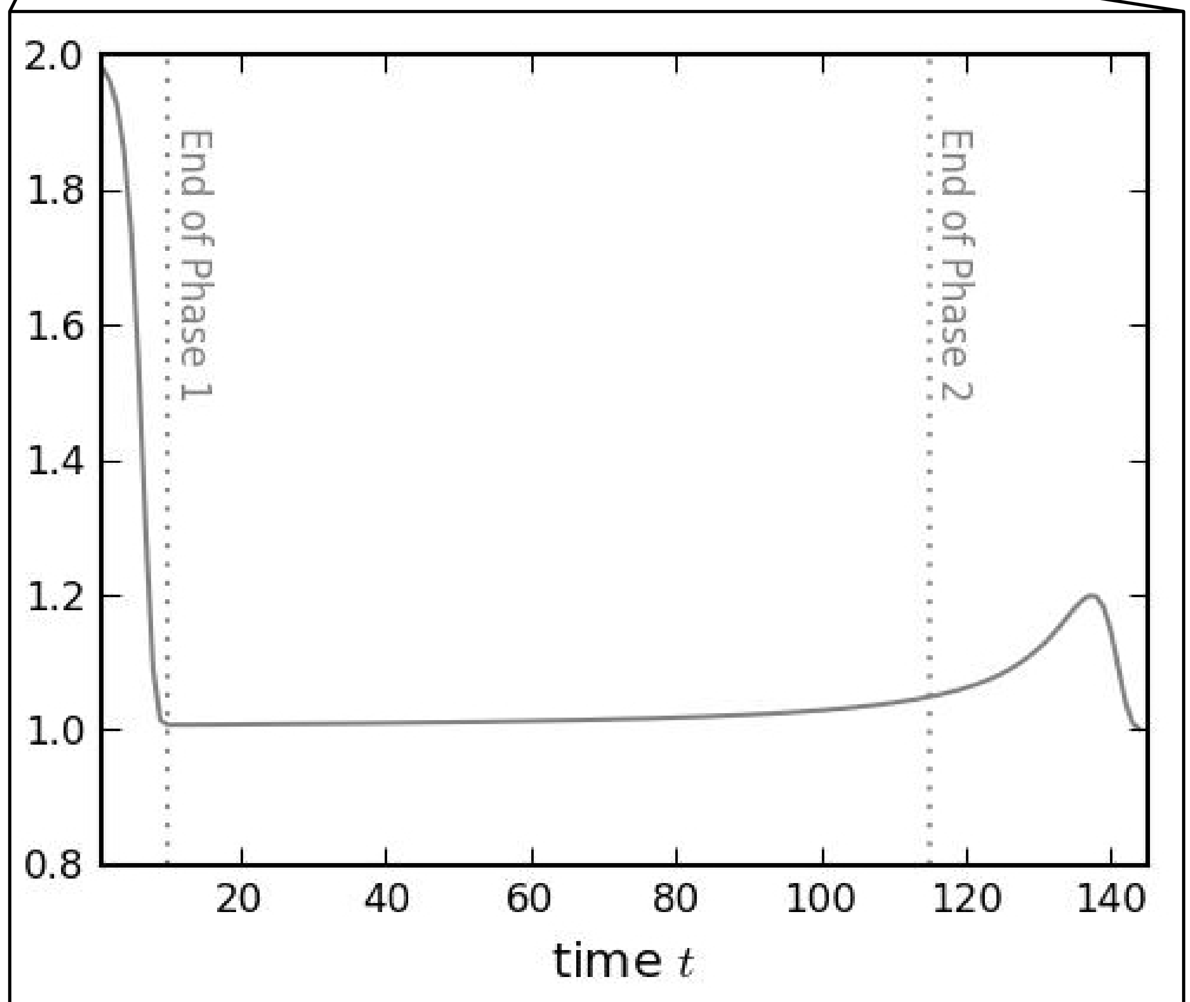
With L. Gualà and S. Leucci: NP-Hardness of Match-3 Games (Candy Crush) [CIG '14]

With D. Kaaser and F. Mallmann-Trenn: Deterministic Binary Majority [brief ann. in DISC '15]

With P. Fraigniaud: "Natural" Consensus with Noisy Communication [Coming soon]

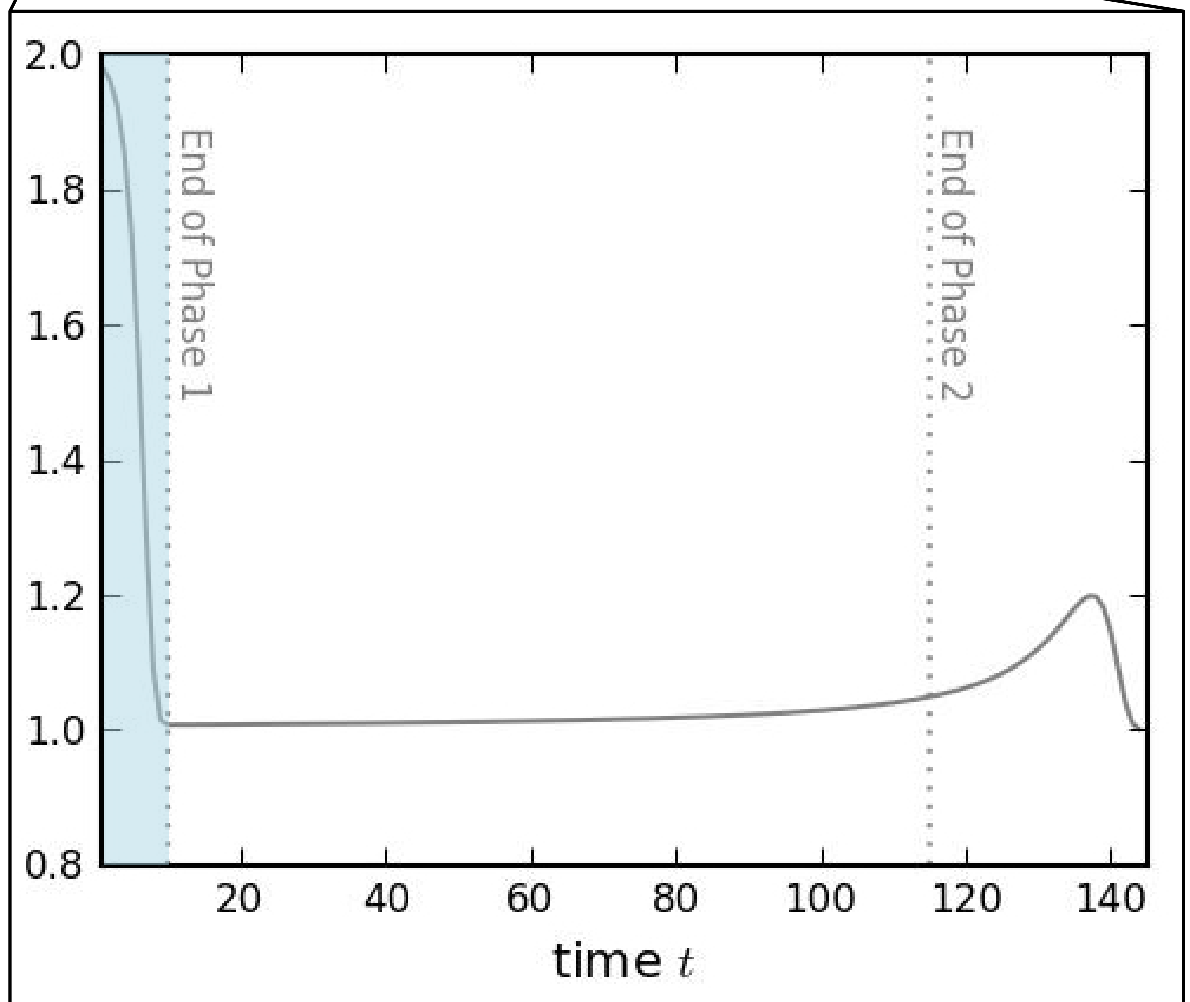
Analysis of Undecided-State

$$\mathbf{E}\left[c_i^{(t+1)} \,\middle|\, \mathbf{c}^{(t)}\right] = c_i^{(t)} \cdot \square$$



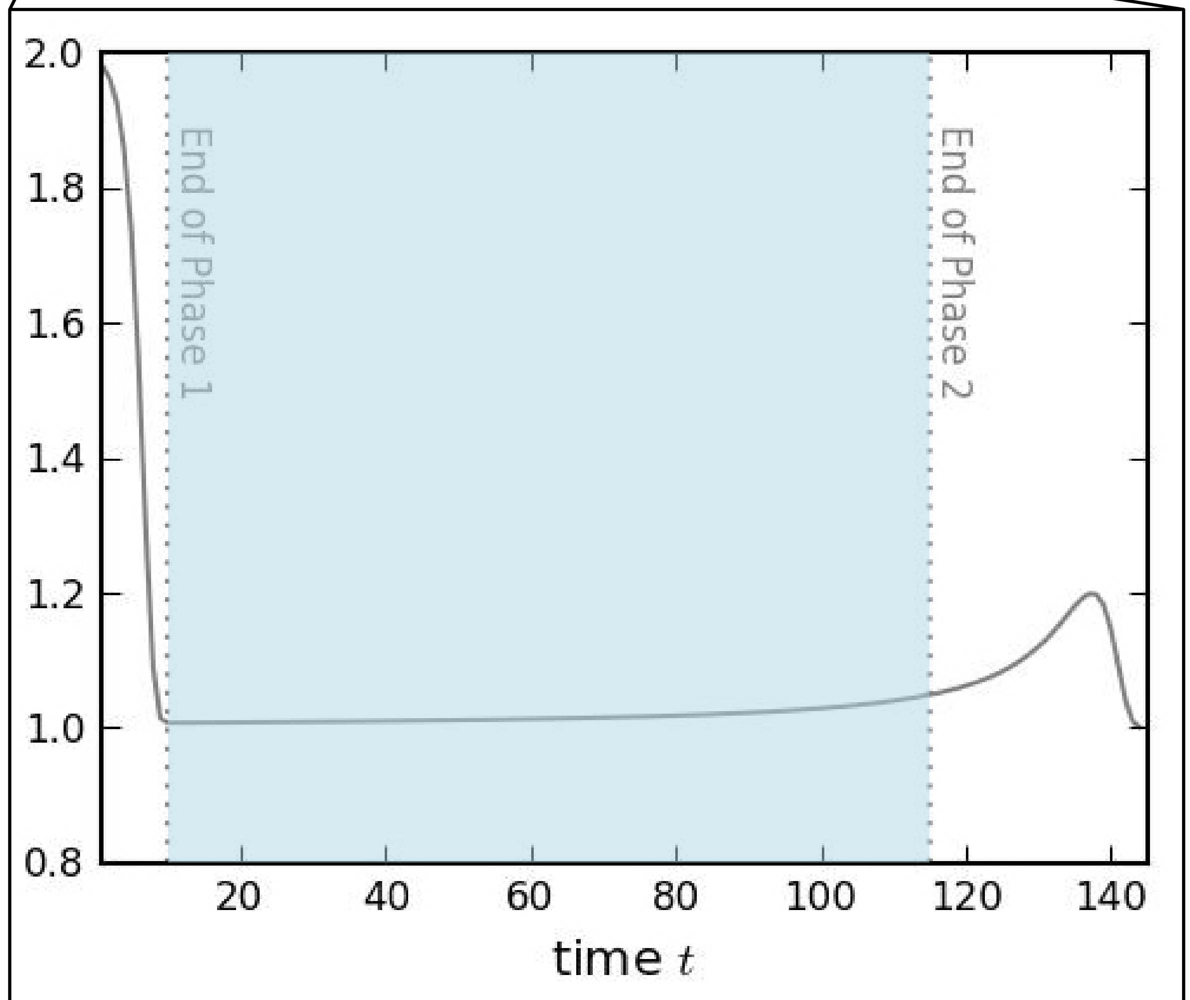
Analysis of Undecided-State

$$\mathbf{E}\left[c_i^{(t+1)} \,\middle|\, \mathbf{c}^{(t)}\right] = c_i^{(t)} \cdot \square$$



Analysis of Undecided-State

$$\mathbf{E}\left[c_i^{(t+1)} \,\middle|\, \mathbf{c}^{(t)}\right] = c_i^{(t)} \cdot \square$$



Analysis of Undecided-State

$$\mathbf{E}\left[c_i^{(t+1)} \,\middle|\, \mathbf{c}^{(t)}\right] = c_i^{(t)} \cdot \square$$