Plurality Consensus in the Gossip Model

Emanuele Natale

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joint work with
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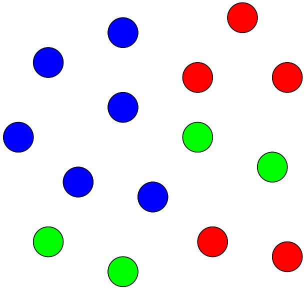
San Diego, 4-6 January 2015

The Plurality Consensus Problem

- We have a set of nodes each having one color out of $\{1, \ldots, k\}$.



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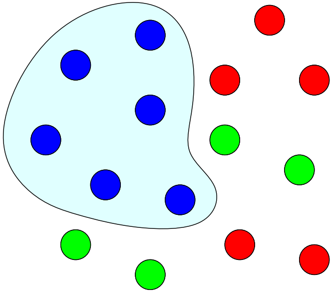

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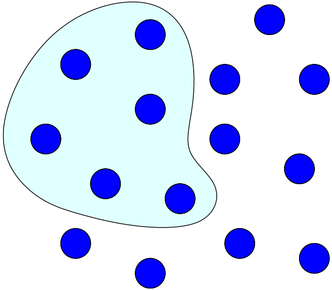
- We have a set of nodes each having one color out of $\{1, \ldots, k\}$.
- There is a plurality of nodes having the same color.
- We want to reach consensus on the plurality color.



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- **Local memory and message size**: $O(\log n)$.

Relationships to Other Communication Models

$\mathcal{GOSSIP}$ model with neighbors chosen randomly: Telephone Call, Push&Pull, Uniform Gossip...

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Every task that can be solved in the $\mathcal{LOCAL}$ model in T rounds, can be solved in $O(T + \text{polylog} n)$ rounds in the $\mathcal{GOSSIP}$ model.
But...

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But... using the preceding theorem, message size grows dramatically!

(Main) Related Works

	Mem. & mess. size	**# of colors**	**Time efficiency**	**Comm. Model**
Kempe et al. FOCS '03	$O(k \log n)$	any	$O(\log n)$	$\mathcal{GOSSIP}$
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Our Contribution: Characterizing the Initial Bias

$$c_i^{(t)} := |\{i\text{-colored nodes}\}|, \quad \text{color 1 is the plurality,}$$

$$q^{(t)} := |\{\underbrace{\text{undecided}}_{\text{...wait 3 slides!}} \text{ nodes}\}|, \quad \mathbf{c}^{(t)} := \left(c_1^{(t)}, \ldots, c_k^{(t)}, q^{(t)}\right)$$

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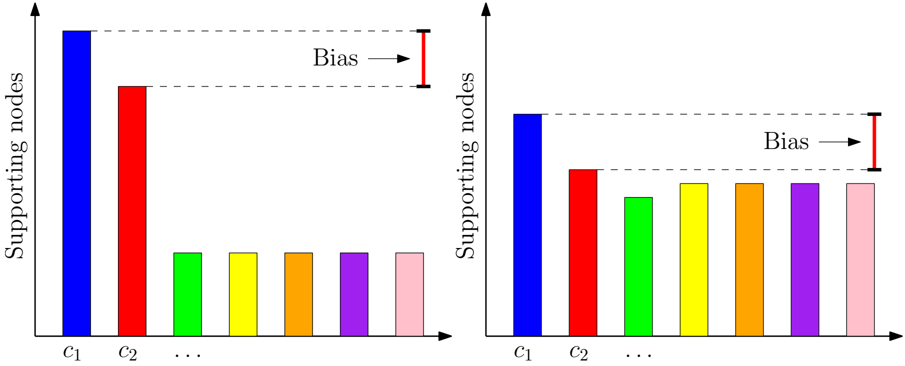
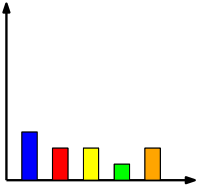


The Monochromatic Distance

$$\mathrm{md}(\mathbf{c}^{(\mathbf{0})}) := \sum_{i=1}^{k} \left(\frac{c_i^{(0)}}{c_1^{(0)}}\right)^2 = 1 + \mathcal{D}\left(\quad , \quad\right)$$



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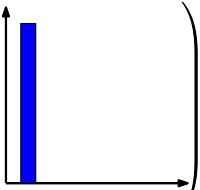

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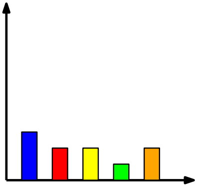
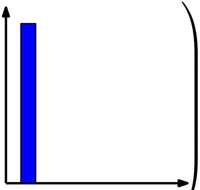
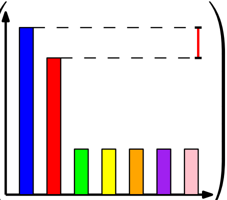
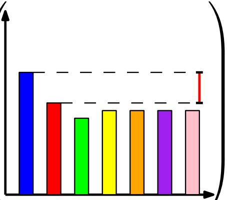




$$1 \leq \mathrm{md}\left(\cdot \right) \ll \mathrm{md}\left(\cdot \right) \leq k$$





Our Results

First analysis for $k = \omega(1)$ of the Undecided-State Dynamics [Angluin et al., Perron et al., Babaee et al., Jung et al.]:

Upper Bound

If $k = O\left((n/\log n)^{1/3}\right)$ and $c_1 \geq (1+\epsilon) \cdot c_2$ with $\epsilon > 0$, then w.h.p. the Undecided-State Dynamics reaches plurality consensus in $O\left(\text{md}(\mathbf{c}^{(0)}) \cdot \log n\right)$ rounds.

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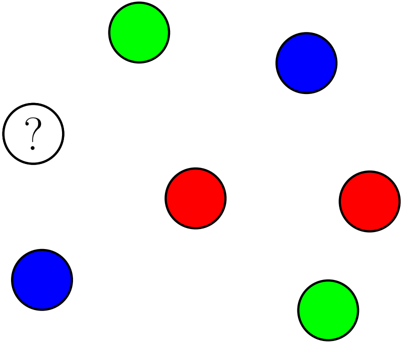
Lower Bound

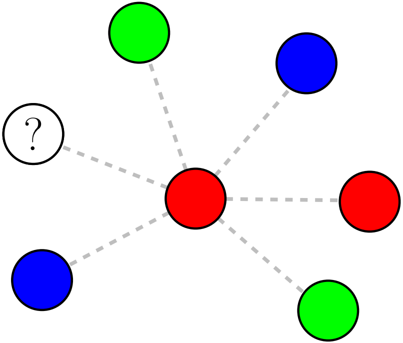
If $k = O\left((n/\log n)^{1/6}\right)$ then w.h.p. the Undecided-State Dynamics converges after at least $\Omega(\mathrm{md}(\mathbf{c}^{(0)}))$ rounds.

The Undecided-State Dynamics



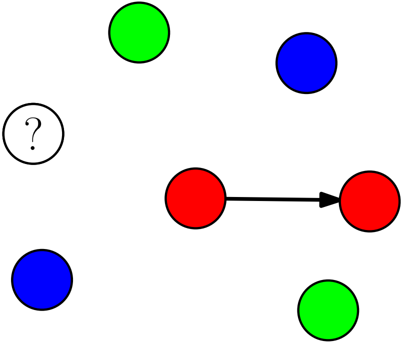
Some nodes can be "undecided".

The Undecided-State Dynamics



At the beginning of each round, each node observes a neighbor picked uniformly at random.

The Undecided-State Dynamics



If the observed node shares the same color...

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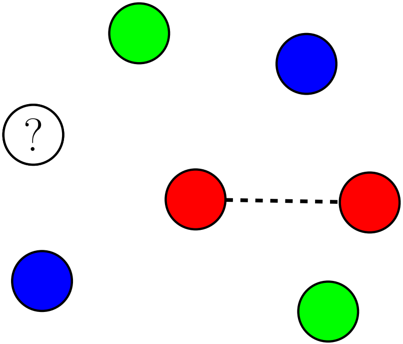
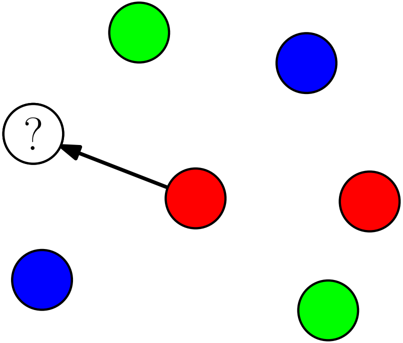


. . . nothing happens;

The Undecided-State Dynamics



if the node observes an undecided one...

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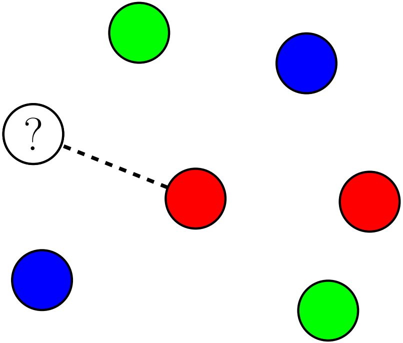
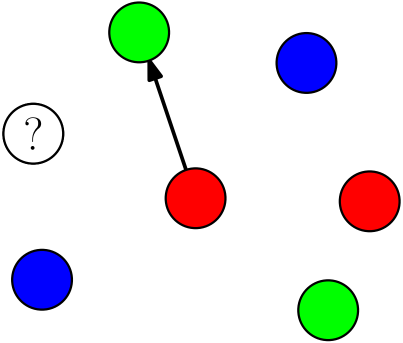


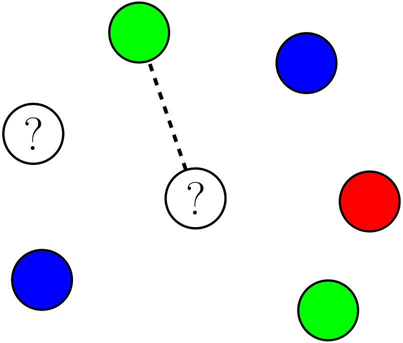
. . . nothing happens too;

The Undecided-State Dynamics



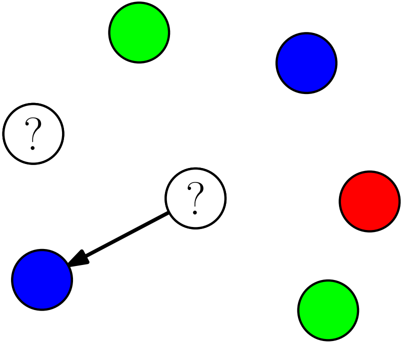
but, if the observed node has a different color. . .

The Undecided-State Dynamics



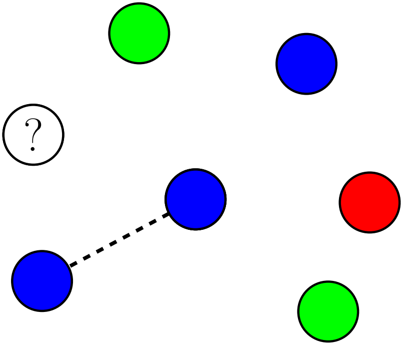
. . . then the node becomes undecided.

The Undecided-State Dynamics



Once undecided. . .

The Undecided-State Dynamics

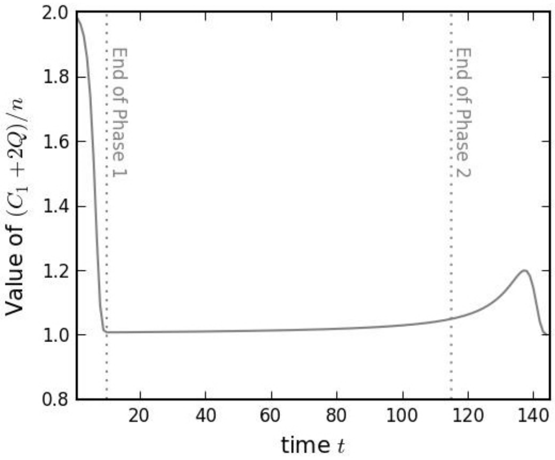


. . . the node copies the first color it sees.

Overview of the Process

$$\mathbf{E}\left[c_i^{(t+1)}\,\middle|\,\mathbf{c}^{(t)}\right] = c_i^{(t)} \cdot \underbrace{\frac{c_i^{(t)} + 2q^{(t)}}{n}}_{\text{Growth factor}}$$

Simulation of the growth factor:



Remarks

W.h.p.:

- Plurality does not change.
- Growth factor of plurality is > 1.

Expected Behaviour of the Process

$$\begin{cases} \mathbf{E}\left[q^{(t+1)} \,\middle|\, \mathbf{c}^{(t)}\right] = \frac{1}{n}\left[\left(q^{(t)}\right)^2 + \left(n - q^{(t)}\right)^2 - \sum_i \left(c_i^{(t)}\right)^2\right] \\ \mathbf{E}\left[c_1^{(t+1)} \,\middle|\, \mathbf{c}^{(t)}\right] = c_1^{(t)} \cdot \frac{c_1^{(t)} + 2q^{(t)}}{n} \\ \qquad \vdots \\ \mathbf{E}\left[c_k^{(t+1)} \,\middle|\, \mathbf{c}^{(t)}\right] = c_k^{(t)} \cdot \frac{c_k^{(t)} + 2q^{(t)}}{n} \end{cases}$$

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Our Key Idea

Tip: Look for $md(\mathbf{c}^{(t)})$ and $R(\mathbf{c}^{(t)}) := \sum_{i=1}^{k} \frac{c_i^{(t)}}{c_1^{(t)}}$.

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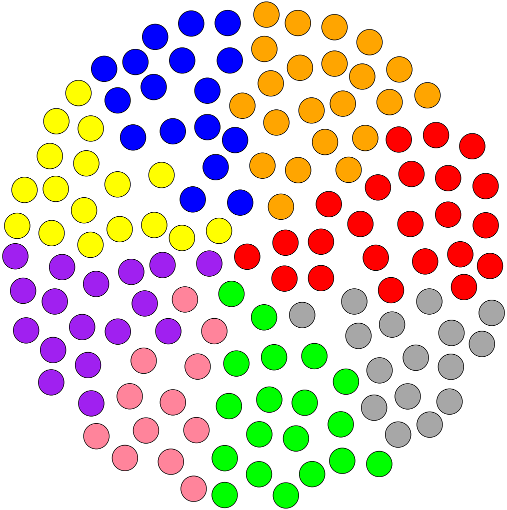
Lemma

$$\mathbf{E}\left[\frac{c_1^{(t+1)} + 2q^{(t+1)}}{n} \middle| \mathbf{c}^{(t)} \right] =$$

$$= 1 + \frac{\left(n - 2q^{(t)} - c_1^{(t)}\right)^2}{n^2} + \frac{2\left(R(\mathbf{c}^{(t)}) - \mathrm{md}(\mathbf{c}^{(t)})\right) \cdot (c_1)^2}{n^2}$$

First Round

Round 1: Each node observes another random one.



First Round

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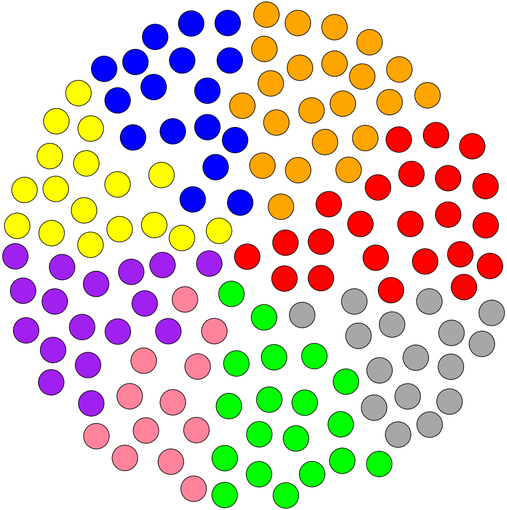
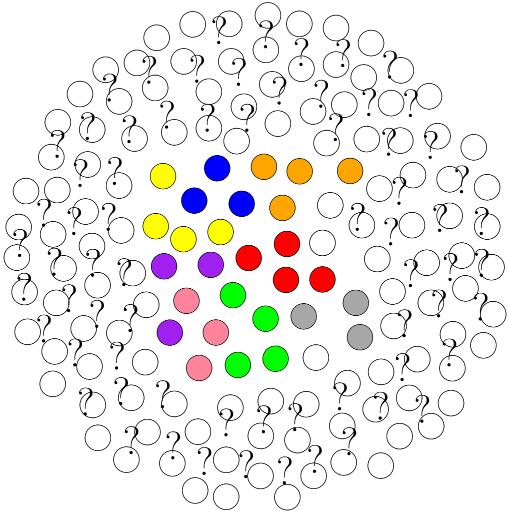
The larger the number of colors and the more uniform the initial distribution, the higher the expected number of undecided nodes.



First Round

The size of each color is reduced to $\frac{(c_i^{(0)})^2}{n}$.



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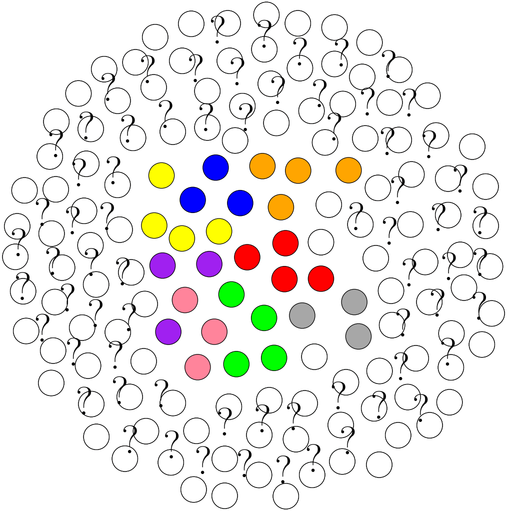
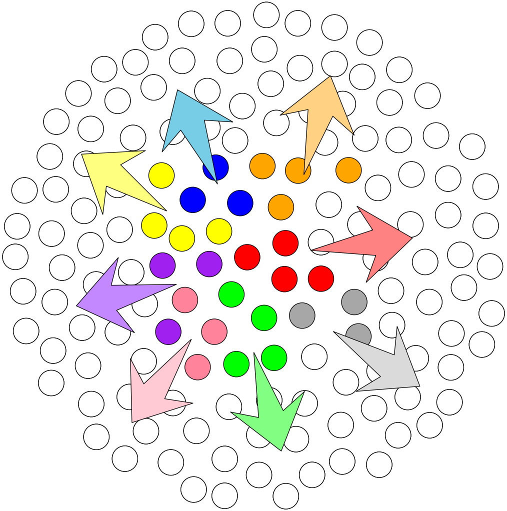
Colors with $c_i^{(0)} = O(\sqrt{n})$ nodes are likely to disappear.



Phase 1

If the initial distribution is quite uniform there are $\Omega(n)$ undecided nodes.

Undecided nodes take the first color they pull, causing colors to spread very fast.



Phase 1

Lemma

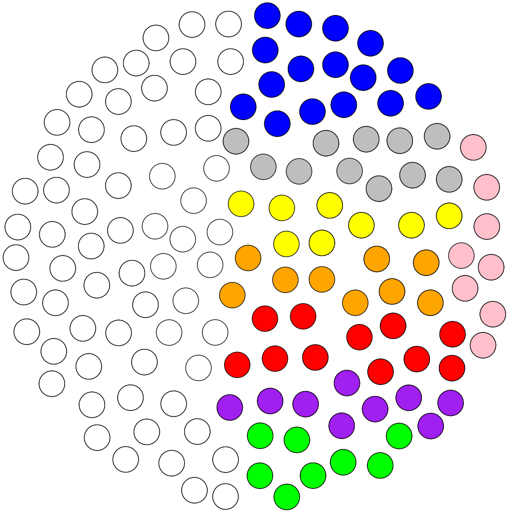
Within $T = O\left(\log \frac{R(\mathbf{c})^2}{\mathrm{md}(\mathbf{c})}\right)$ rounds the system reaches a configuration such that w.h.p.

$$c_1^{(T)} = \Theta\left(\frac{n}{\mathrm{md}(\mathbf{c})}\right)$$
$$q^{(T)} = \frac{n}{2}\left(1 \pm \Theta\left(\frac{1}{\mathrm{md}(\mathbf{c})}\right)\right)$$

and, for every i, $c_1^{(0)}/c_i^{(0)}$ is approximately preserved.

Phase 2

new colored
$\approx$
new undecided.



Phase 2

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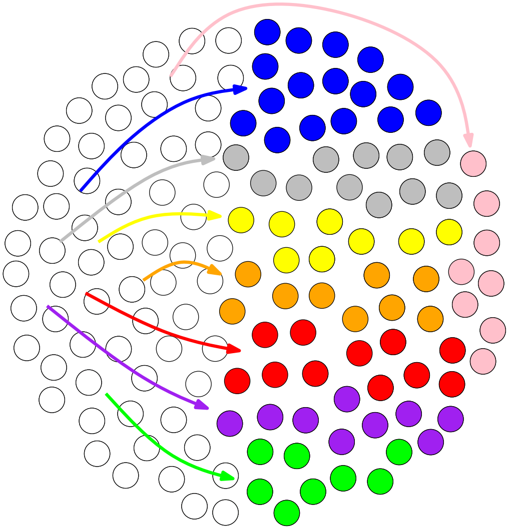

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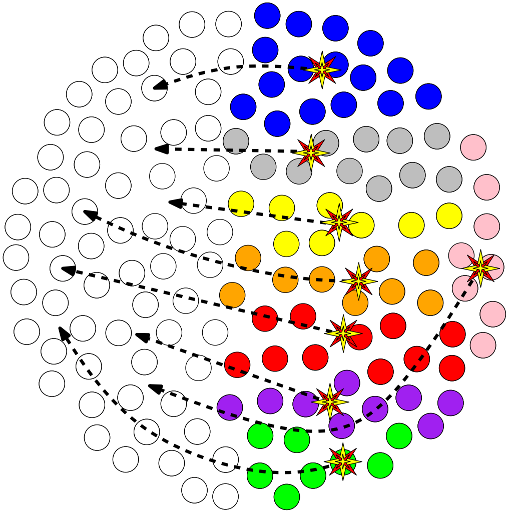

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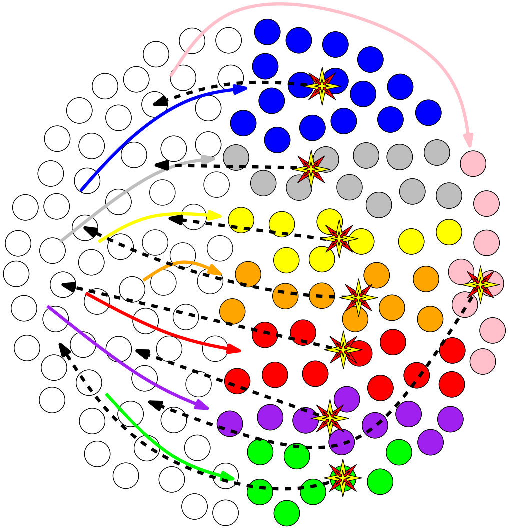
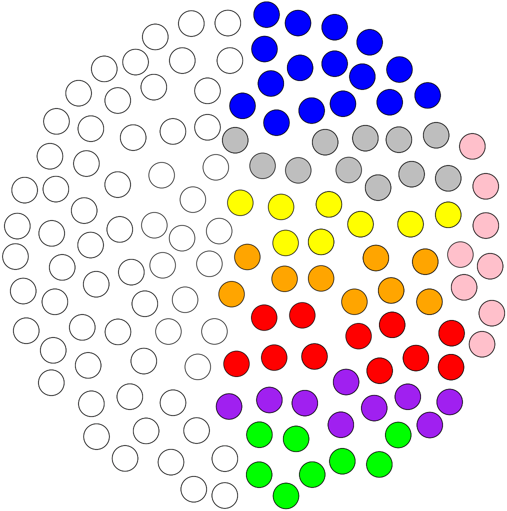


Phase 2

The plurality has a small advantage $\Longrightarrow$ after long time the equilibrium breaks down.



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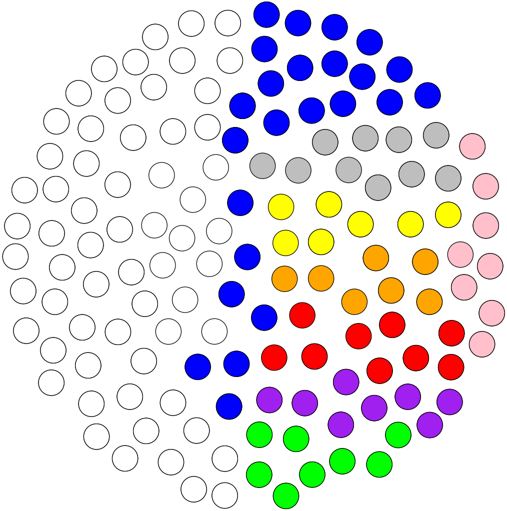

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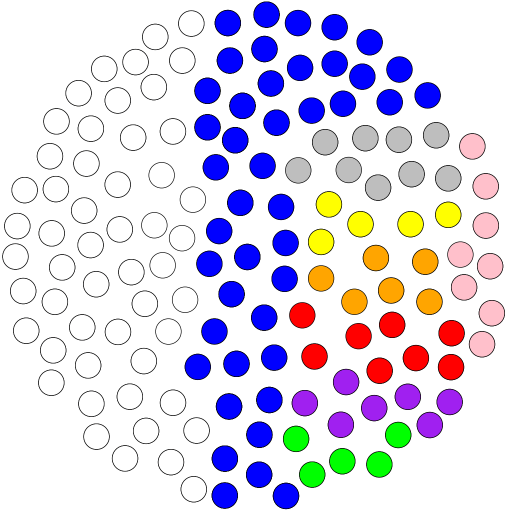

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$$\text{Plateau around} \begin{cases} c_1^{(T)} & = \Theta\left(\frac{n}{\mathrm{md}(\mathbf{c})}\right) \\ q^{(T)} & = \frac{n}{2}\left(1 \pm \Theta\left(\frac{1}{\mathrm{md}(\mathbf{c})}\right)\right) \end{cases}$$

Average Growth:

$$\mathbf{E}\left[c_1^{(t+1)} \,\middle|\, \mathbf{c}^{(t)}\right] \approx c_1^{(t)}\left(1 + \Theta\left(\frac{1}{\mathrm{md}(\mathbf{c})}\right)\right)$$

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$\implies$ Lower bound of $\Omega\left(\mathrm{md}(\mathbf{c})\right)$.

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$\implies$ After $O(\mathrm{md}(\mathbf{c}) \log n)$ rounds, $R(\mathbf{c}^{(t)}) = 1 + o(1)$.

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$\implies$ Plurality Consensus is reached within $O(\log n)$ rounds.

Extension to d-Regular Expanders

Given a d-regular expander graph, $k = O\left((n/\log n)^{1/3}\right)$ and $c_1 \geq (1+\epsilon) \cdot c_2$ with $\epsilon > 0$, using polylogarithmic memory and message size the plurality consensus problem can be solved in w.h.p. $O(\text{md}(\mathbf{c})\text{polylog}(n))$ rounds.

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- The $\mathcal{GOSSIP}$ model with $O(\mathrm{polylog}\, n)$ limit on message size: congestion when random walks meet.

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